

Solution to extra exercises

1. a) 120 minutes $\Rightarrow$ 4 per 2 seconds

$$\Rightarrow X \sim \text{Poiss}(4)$$

$$P(X < 4) = P(X \leq 3) = 0.433 \quad (\text{by the table})$$

b) 6 calls per 3 seconds

$$X \sim \text{Poiss}(6)$$

$$\begin{aligned} P(X \geq 3) &= 1 - P(X < 3) \\ &= 1 - P(X \leq 2) = 1 - 0.062 = 0.938 \quad (\text{table}) \end{aligned}$$

c) Let Y be the random variable giving the time of the first call. Y follows an exponential distribution of parameter $\lambda = 2$ (2 calls per second).

$$P(Y > 3) = \int_3^{\infty} \lambda e^{-\lambda t} dt = -e^{-\lambda t} \Big|_3^{\infty} = e^{-3\lambda} = e^{-6}$$

2. Let X be the random variable giving the number of males. X follows a binomial distribution with $n=6$ and $p=0.4$.

More male than female means that X should be greater than 3

$$P(X > 3) = 1 - P(X \leq 3) = 1 - 0.8208 = 0.1792$$

3. a) Numbers that are multiple of 3;

3 - 6 - 9 - 12 - 15 - 18

$$p(\text{multiple of } 3) = \frac{6}{20}$$

$$b) \text{Geom}\left(\frac{6}{20}\right) \quad p(X=7) = \left(\frac{14}{20}\right)^6 \cdot \frac{6}{20}$$

4. a) Binomial with $n=10$, $p=0,4$

$$P(X=2) = \binom{10}{2} (0,4)^2 (0,6)^8$$

b) Geometric with $p=0,4$

$$P(X=5) = (0,6)^4 \cdot 0,4$$

c) Negative binomial with $r=3$ and $p=0,4$

$$P(X=10) = \binom{9}{2} (0,4)^3 (0,6)^7$$

5. Hypergeometric. $N=100$

$$P(X=3) = \frac{\binom{35}{3} \binom{65}{7}}{\binom{100}{10}}$$

6. $Y = \text{"test says yes"}$

$A = \text{"person has an allergy"}$

$$P(Y|A) = 0,8 \quad P(Y|A') = 0,1 \quad P(A) = 0,01$$

$$\text{Find } P(A|Y) = \frac{P(Y|A) \cdot P(A)}{P(Y|A) \cdot P(A) + P(Y|A') \cdot P(A')} = \frac{0,8 \cdot 0,01}{0,8 \cdot 0,01 + 0,1 \cdot 0,99} = \frac{0,008}{0,107} = 0,075$$

7. I a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0,1 + 0,2 - 0,05 = 0,25$$

b) $P(A) - P(A \cap B) = 0,1 - 0,05 = 0,05$

c) $P(A' \cap B') = 1 - P(A \cup B) = 0,75$

d) $P(A \cup B) - P(A \cap B) = 0,25 - 0,05 = 0,2$

II. $P(A) \cdot P(B) = 0,1 \cdot 0,2 = 0,02 \neq 0,05$

$$P(A \cap B) \neq P(A) \cdot P(B) \Rightarrow \text{not independent}$$