Some exercises week 1 MSA101/MVE187 2022

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August 29, 2022

1. Tests with the outcomes "success" or "failure" are done, each with a probability of success θ, and their outcomes are indpendent given θ. The prior for θ is uniform on the interval from zero to one. Assume 5 tests are done, and 4 are successful. What is the posterior for θ given this information? What is the probability that θ is 0.8 or more?

2. Each day, a count of some rare birds are made. We assume the counts are Poisson distributed with parameter λ, and that they are independent given λ. The prior for λ is Gamma$(3, 1)$

   (a) In three consecutive days, 5, 7, and 4 birds are counted. What is the posterior for λ given this information?

   (b) What is the predictive probability distribution for the number of birds that will be counted on the fourth day? What is the probability that at least 5 birds will be counted?

3. We assume observations are sampled from a normal distribution Normal$(3, 1/\tau)$, where τ has a Gamma$(2, 2)$ prior. The values observed are 3.5, 4.1, and 2.3.

   (a) Find the posterior distribution for τ given the data.

   (b) Find the probability density for the predictive distribution for the next observation that is sampled from the distribution. Can you recognize this as a distribution in a named family of distributions?