

$$X_1, X_2, \dots, X_n \sim \text{NORMAL}(\mu, 1)$$

HOW DO YOU LEARN ABOUT μ FROM DATA $X_1, \dots, X_n$?

NEED AN ESTIMATOR FOR μ

$$\bar{X} = \frac{1}{n} (X_1 + \dots + X_n) \quad \text{ESTIMATES } \mu$$

"FIND THE UNCERTAINTY" BY STUDYING THE ESTIMATOR

$\bar{X}$ IS RANDOM VARIABLE

$$E(\bar{X}) = E\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n} \sum_{i=1}^n E(X_i) = \frac{1}{n} \cdot n\mu = \mu$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n^2} \sum_{i=1}^n \text{Var}(X_i) = \frac{1}{n^2} \cdot n \cdot 1 = \frac{1}{n}$$

$$\bar{X} \sim \text{NORMAL}\left(\mu, \frac{1}{n}\right)$$

$$\text{WANT: } \mu \sim \text{NORMAL}\left(\bar{X}, \frac{1}{n}\right)$$