HOW CAN WE FIX THIS?

$X_1, \ldots X_n \sim \text{NORMAL}(\mu, 1)$ GOT: $\bar{X} \sim \text{NORMAL}(\mu, 1/n)$

WANT $\mu \sim \text{NORMAL}(\bar{x}, 1/n)$

- μ NEEDS TO BE A RANDOM VARIABLE
- DEFINE A JOINT DISTRIBUTION (DENSITY) ON $x_1, x_2 \ldots x_n, \mu$:

$X_i | \mu \sim \text{NORMAL}(\mu, 1)$

$\mu | x_1, \ldots x_n \sim \text{NORMAL}(\bar{x}, 1/n)$

PROPOSE: DEFINE A JOINT PROBABILITY DENSITY

$$\pi(x_1, x_2 \ldots x_n, \mu) = \prod_{i=1}^{n} \text{NORMAL}(x_i; \mu, 1) \qquad \pi(\mu) = 1$$

$$\text{NORMAL}(x_i; \mu, 1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_i - \mu)^2\right)$$