

$$\pi(\theta|y) \propto_{\theta} \pi(y|\theta) \pi(\theta)$$

$$= e^{-\theta} \frac{\theta^y}{y!} \cdot \frac{\beta^{\alpha}}{\Gamma(\alpha)} \theta^{\alpha-1} \exp(-\beta\theta)$$

$$\propto_{\theta} \theta^{\alpha+y-1} \exp(-\beta\theta - \theta)$$

$$\propto_{\theta} \text{GAMMA}(\theta; \alpha+y, \beta+1)$$