

$$\pi(y_i) = \frac{\pi(y_i | \lambda) \pi(\lambda)}{\pi(\lambda | y_i)}$$

$$= \frac{\text{EXPONENTIAL}(y_i; \lambda) \text{Gamma}(\lambda; \alpha, \beta)}{\text{Gamma}(\lambda; \alpha + 1, \beta + y_i)}$$

$$= \frac{\lambda \exp(-\lambda y_i) \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} \exp(-\beta \lambda)}{\frac{(\beta + y_i)^{\alpha+1}}{\Gamma(\alpha+1)} \lambda^\alpha \exp(-(\beta + y_i) \lambda)}$$

$$= \frac{\beta^\alpha \Gamma(\alpha+1)}{(\beta + y_i)^{\alpha+1}} = \frac{\alpha}{\beta} \left(\frac{\beta}{\beta + y_i} \right)^{\alpha+1}$$