Algorithms. Assignment 7

Problem 12

Some definitions first: A Boolean *term* consists of literals connected by $\wedge$, and a *disjunctive normal form (DNF)* consists of terms connected by $\vee$. Informally speaking, a DNF is a Boolean formula that looks like a CNF, however the roles of $\wedge$ and $\vee$ are switched.
Now consider the following reasoning:

"The SAT problem for DNF is easy to solve: It suffices to make one term true. Unless every term contains some pair of a variable and its negation, we can always pick some term and make it true, simply by setting all literals in this term true.
Furthermore, we can easily transform every CNF into an equivalent DNF, using the distributive laws, e.g.: $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$. Therefore, the SAT problem for CNF is easy to solve as well.
On the other hand, we know that SAT for CNF is NP-complete. This implies P=NP, which solves the famous P-NP-problem."

Where exactly is the flaw in this reasoning? Explain your objection in detail.

Problem 13

In the Subset Sum problem we are given n positive integers w_i and another positive integer W, and the problem is to decide whether some subset has a sum exactly W. Subset Sum is NP-complete; you can use this statement without proof.
Now we define the Half-Half Subset Sum problem, as the special case of the Subset Sum problem where $W = \sum_{i=1}^{n} w_i/2$. (In words: Can we split the given set of integers half-half?)

Prove that Half-Half Subset Sum is still NP-complete. Specifically:
Give a reduction from the "unrestricted" Subset Sum problem and describe all necessary ingredients: How does your reduction work? Does it run in polynomial time? Show that the produced instance of Half-Half Subset Sum is equivalent to the given instance of Subset Sum.
But do not think complicated. It suffices to insert one extra item of suitable size in the given instance of Subset Sum ...