In order to get hom. BC for v, we observe that

$1 \overset{!}{=} \underset{(H)}{u(0,t)} \underset{\text{Def } u}{=} v(0,t) + s(0)$ and $3 \overset{!}{=} \underset{(H)}{u(1,t)} \underset{\text{Def } u}{=} v(1,t) + s(1)$

Hence, one sets $s(0) \overset{!}{=} 1$ and $s(1) \overset{!}{=} 3$ in order to get

$$v(0,t) = 0 = v(1,t)$$

In order to get IC, we observe that

$f(x) \underset{(H)}{=} u(x,0) \underset{\text{Def } u}{=} v(x,0) + s(x)$ or $v(x,0) \overset{!}{=} f(x) - s(x)$

↳ We found 2 "easier" problems:

$$\begin{cases} v_t - v_{xx} = 0 \\ v(0,t) = v(1,t) = 0 \\ v(x,0) = f(x) - s(x) \end{cases} \qquad \begin{cases} -s''(x) = 8 \\ s(0) = 1,\ s(1) = 3 \end{cases}$$

hom. lin. PDE → ok :-) BVP ⇝ ok :-)

Sol. to BVPs

$-s''(x) = 8 \Rightarrow s(x) = Ax^2 + Bx + C$ with $s'(x) = 2Ax + B$ and $s''(x) = 2A$

Hence $2A = -8$ or $\boxed{A = -4}$

$s(0) = 1 \Rightarrow \boxed{C = 1}$ $\quad s(1) = 3 \Rightarrow 3 = -4 + B + 1 \Rightarrow \boxed{B = 6}$

This gives $s(x) = -4x^2 + 6x + 1$ //

Sol. to PDE:

$v_t - v_{xx} = 0$

$v(0,t) = v(1,t) = 0$

$v(x,0) = f(x) - s(x) = f(x) - (-4x^2 + 6x + 1)$

Use separation of variables $v(x,t) = X(x) \cdot T(t)$ and Fourier series as in the previous section to find $v(x,t)$

Answer: $u(x,t) = v(x,t) + (-4x^2 + 6x + 1)$ //

The end ?