Basic rules for differentiation and integration
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Basic rules for differentiation

Under the usual assumptions, one has:

- Derivative of a sum

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)+g(x)\big)=\frac{\mathrm{d}}{\mathrm{d}x}f(x)+\frac{\mathrm{d}}{\mathrm{d}x}g(x)=f'(x)+g'(x).$$

- Derivative with a constant factor $c\in\mathbb{R}$

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big)=c\frac{\mathrm{d}}{\mathrm{d}x}f(x)=cf'(x).$$

- Derivative of a product

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)g(x)\big)=f'(x)g(x)+g'(x)f(x).$$

- Derivative of a quotient

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}.$$

- Chain rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\big((f\circ g)(x)\big)=f'(g(x))g'(x).$$

Basic derivatives

Under appropriate assumptions

$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
$\ln(x)$	$1/x$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\arcsin(x)$	$1/\sqrt{1-x^2}$
$\arccos(x)$	$-1/\sqrt{1-x^2}$
$\arctan(x)$	$1/(1+x^2)$

Lists of derivatives can be found under wikipedia.org.

Basic rules for integration

Under the usual assumptions, one has:

- Integral of a sum
$$\int \big(f(x)+g(x)\big)\,\mathrm{d}x = \int f(x)\,\mathrm{d}x + \int g(x)\,\mathrm{d}x.$$
- Integral with a constant factor $c \in \mathbb{R}$
$$\int \big(cf(x)\big)\,\mathrm{d}x = c\int f(x)\,\mathrm{d}x.$$
- Integration by parts
$$\int f(x)g'(x)\,\mathrm{d}x = \Big[f(x)g(x)\Big] - \int f'(x)g(x)\,\mathrm{d}x.$$
- Do not forget the method of substitution.

Basic integrals

Under appropriate assumptions

$f(x)$	$\int f(x)\,\mathrm{d}x$
e^x	e^x
x^n for $n \neq -1$	$x^{n+1}/(n+1)$
$1/x$	$\ln\lvert x\rvert$
a^x	$a^x/\ln(a)$
$\cos(x)$	$\sin(x)$
$\sin(x)$	$-\cos(x)$
$\tan(x)$	$-\ln\lvert\cos(x)\rvert$

Warning. Do not forget the constant of integration!
Lists of integrals can be found under wikipedia.org.

Examples

Compute the following derivatives (the answers are given)

1. $(5)' = \ldots = 0$.
2. $(-2x^3)' = \ldots = -6x^2$.
3. $((3x^2+1)^3)' = \ldots = 18x(3x^2+1)^2$.
4. $(\sin(x^2+x))' = \ldots = \cos(x^2+x)(2x+1)$.

5. $(\frac{x^2-1}{3x^4})' = \ldots = \frac{6x^5-12(x^2-1)x^3}{9x^8}$.
6. $((2x^5-x)(3x+1))' = \ldots = 36x^5+10x^4-6x-1$.
7. $(\sqrt{1-x^2})' = \ldots = -\frac{x}{\sqrt{1-x^2}}$.
8. $(e^{2x^2+3x})' = \ldots = (4x+3)e^{2x^2+3x}$.
9. $(4^x)' = \ldots = \ln(4)4^x$.

Compute the following integrals (the answers are given)

1. $\int 2x\,dx = \ldots = x^2 + C$.
2. $\int \cos(2x)\,dx = \ldots = \frac{1}{2}\sin(2x) + C$.
3. $\int e^{-3x}\,dx = \ldots = -\frac{1}{3}e^{-3x} + C$.
4. $\int (x+3)^2\,dx = \ldots = \frac{1}{3}x^3 + 3x^2 + 9x + C$.
5. $\int x\ln(x)\,dx = \ldots = \frac{x^2}{2}\ln(x) - \frac{1}{4}x^2 + C$.
6. $\int x^2e^x\,dx = \ldots = x^2e^x - 2xe^x + 2e^x + C$.
7. $\int \frac{1}{7x}\,dx = \ldots = \frac{1}{7}\ln|x| + C$.
8. $\int (1+\tan^2 x)\,dx = \ldots = \tan(x) + C$.
9. $\int \sin(x)\cos(x)\,dx = \ldots = \frac{\sin^2(x)}{2} + C$.