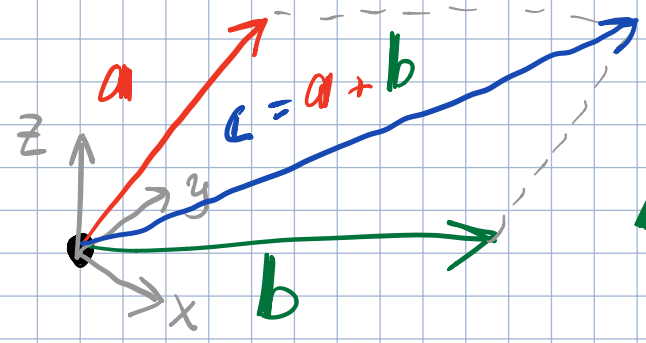
Vectors in 3D

$$a = (a_1, a_2, a_3)$$



x y z

$$b = (b_1, b_2, b_3)$$

$$c = (c_1, c_2, c_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

Example:

$$a = (1, -1, 2) \qquad b = (-2, 0, 1)$$

$$c = (1-2, -1+0, 2+1) = (-1, -1, 3)$$

a t

$$ta = (ta_1, ta_2, ta_3)$$ multiplication with scalar

Length of a vector:

$\|a\|$ or $|a|$ ← notation for length of vector

also called the "norm" of a

$$\|a\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Try to compute $\|ta\|$

$$\|ta\| = \sqrt{(ta_1)^2 + (ta_2)^2 + (ta_3)^2}$$

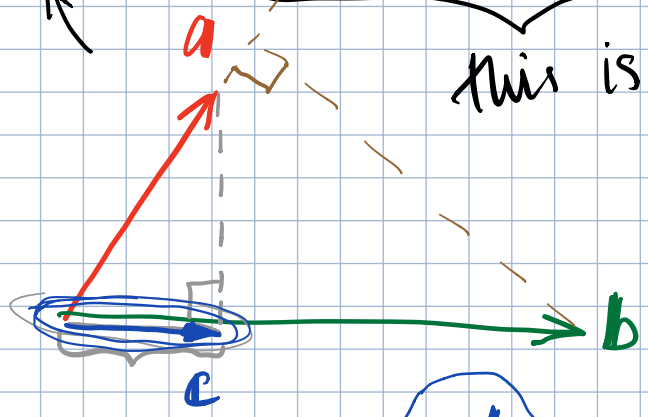
$$= \sqrt{t^2(a_1^2 + a_2^2 + a_3^2)} = \underbrace{\sqrt{t^2}}_{t} \cdot \underbrace{\sqrt{a_1^2 + a_2^2 + a_3^2}}_{\|a\|}$$

$$= t\|a\|$$

Dot product of vectors (scalar product)

$$a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$$

this is a scalar!

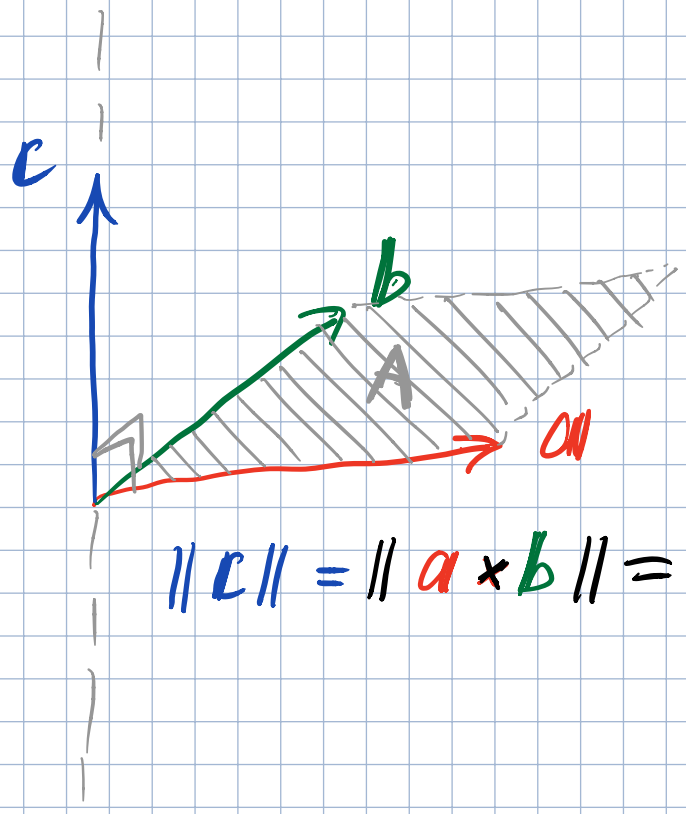


$$c = sb = \frac{a \cdot b}{\|b\|^2} b$$

s

Cross product

$$\mathbf{a} \times \mathbf{b} = (a_2 b_3 - b_2 a_3,\ a_3 b_1 - b_3 a_1,\ a_1 b_2 - b_1 a_2) = \mathbf{c}$$



$$\|\mathbf{c}\| = \|\mathbf{a} \times \mathbf{b}\| = A$$