MVE560 Architectural Geometry, Lecture 1

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Mathematical Sciences
CHALMERS

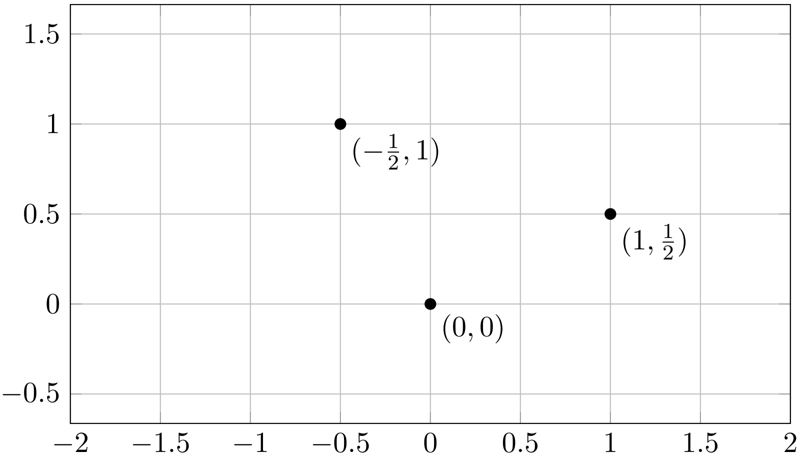
Outline

The Cartesian Coordinate System

- The ‘usual’ coordinate system we use most of the time in $\mathbb{R}^n$
- Named after French philosopher René Descartes (1596–1650)
- Orthogonal/perpendicular *coordinate axes* — the x-axis and the y-axis (and sometimes the z-axis)
- The *origin* is a special ‘reference point’ with coordinates $(0, 0)$ (or $(0, 0, 0)$ if in 3D)
- May be used for both points and vectors

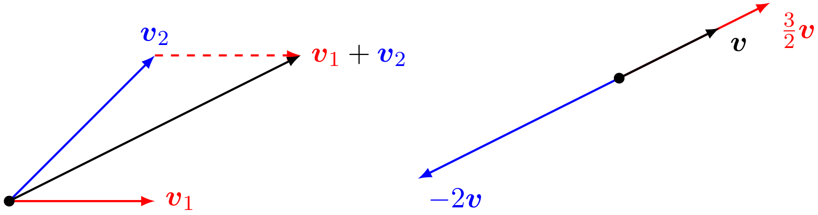
Image source: Wikipedia

The Cartesian Coordinate System — Illustration



Vectors

Vectors live in a *vector space* V (in our case typically $V = \mathbb{R}^2$ or $V = \mathbb{R}^3$), equipped with the operations *addition* and *scaling*:



A vector is best thought of as *motion* or a *direction*.

Vectors in Cartesian Coordinates

In Cartesian coordinates, vector addition and scaling works as follows:

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

and

$$\lambda(x, y, z) = (\lambda x, \lambda y, \lambda z).$$

The length, or *norm*, of a vector $\boldsymbol{v} = (x, y, z)$ is given by the Pythagorean theorem:

$$\|\boldsymbol{v}\| = \sqrt{x^2 + y^2 + z^2}.$$

Points and Vectors

Both vectors and points are often represented using coordinates, e.g. (x, y, z), but they are conceptually *very* different!

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- $[\text{point}] + [\text{vector}] = [\text{point}]$
- ...

Lines

A *line* ℓ consists of all points which can be reached by starting out in a point $\boldsymbol{p}_0$ and going in the direction given by a vector $\boldsymbol{v} \neq \mathbf{0}$:

$$\ell : \boldsymbol{p}(\lambda) = \boldsymbol{p}_0 + \lambda \boldsymbol{v}.$$

There exists exactly one line through two distinct points $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$:

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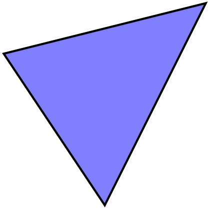
The *line segment* between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ is given by

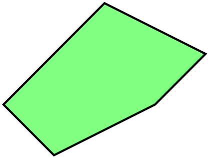
$$\ell : \boldsymbol{p}(\lambda) = \boldsymbol{p}_1 + \lambda(\boldsymbol{p}_2 - \boldsymbol{p}_1), \quad 0 \leq \lambda \leq 1.$$

Triangles (and Other Polygons)

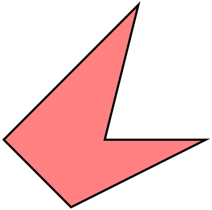
An n-sided *polygon* is a planar object consisting of *vertices* (corners) which are connected in a particular order by *edges* (line segments):



Triangle



Convex polygon



Non-convex polygon

A polygon is called *convex* if it has no inward ‘dents’.

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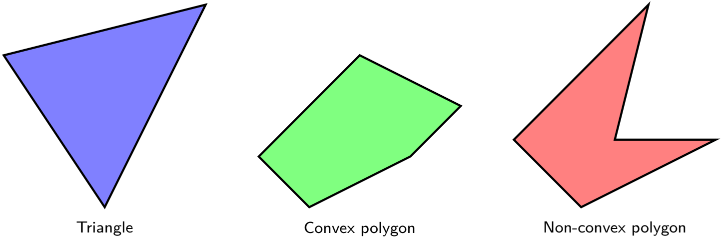
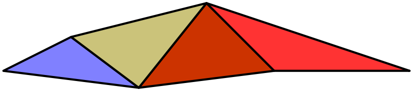


A polygon is called *convex* if it has no inward 'dents'.

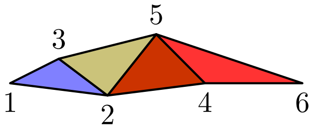
Triangles are always convex and planar, and are therefore the polygon most often used to construct things!

Triangle Meshes



- A *triangle mesh* is a surface consisting of a number of triangles which are joined along their edges.
- With sufficiently many and sufficiently small triangles, triangular meshes can approximate most shapes very well!

Representing Triangle Meshes



A triangle mesh is often represented using a *vertex list*

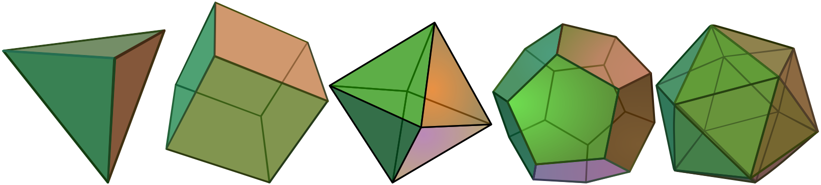
$$V = \begin{bmatrix} x_1 & x_2 & \cdots & x_6 \\ y_1 & y_2 & \cdots & y_6 \\ z_1 & z_2 & \cdots & z_6 \end{bmatrix}$$

and a *triangle list*

$$T = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 4 & 5 & 6 \\ 3 & 5 & 3 & 5 \end{bmatrix},$$

where the indices of the vertices are entered anticlockwise.

The Platonic Solids



Tetrahedron Hexahedron Octahedron Dodecahedron Icosahedron

Image source: Wikipedia

- The five *Platonic solids* shown are the only convex solids whose faces are regular polygons
- Many fascinating properties, i.e. symmetries, relations, ...

Sutton, *Platonic & Archimedean Solids*, 2002.

Cylindrical Coordinates

Cylindrical coordinates (r, φ, z) are related to Cartesian coordinates as

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z. \end{cases}$$

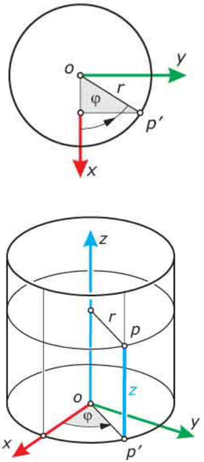


Image source: Pottmann *et al.*

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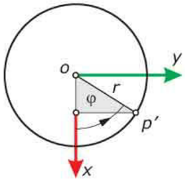
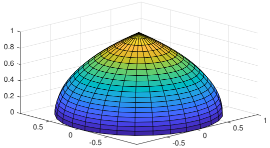
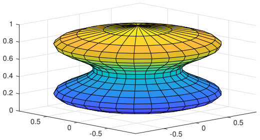
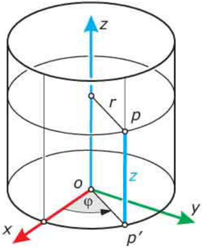


Cylindrical coordinates are very useful for describing various kinds of rotational symmetries:







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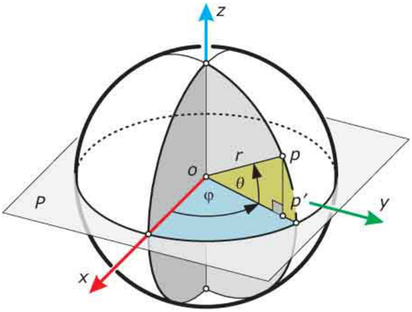
Spherical Coordinates

Spherical coordinates (r, φ, θ) are related to Cartesian coordinates as

$$\begin{cases} x = r\cos\varphi\cos\theta \\ y = r\sin\varphi\cos\theta \\ z = r\sin\theta. \end{cases}$$



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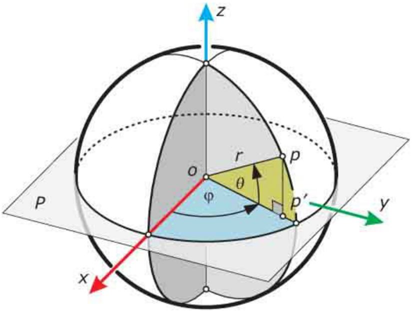
Spherical coordinates are useful for 'placing' things in space, e.g. positioning other geometric primitives.



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