

$$P_n(y_{\text{new}} = 1 | y) = \int_0^1 P_n(y_{\text{new}} = 1 | \theta) \pi(\theta | y) d\theta$$

$$= \int_0^1 \theta \text{Beta}(\theta; y+1, n-y+1) d\theta = \frac{y+1}{(y+1) + (n-y+1)}$$

EXPECTATION OF THIS $\text{Beta}(\alpha, \beta)$ IS $\frac{\alpha}{\alpha + \beta}$

$$\underline{\underline{\frac{y+1}{n+2}}}$$