

$$\pi(\theta|x) \propto_{\theta} \pi(x|\theta) \pi(\theta)$$

$$= e^{-\theta} \frac{\theta^x}{x!} \cdot \frac{\beta^{\alpha}}{\Gamma(\alpha)} \theta^{\alpha-1} \exp(-\beta\theta)$$

$$\propto_{\theta} e^{-\theta} \theta^x \theta^{\alpha-1} e^{-\beta\theta}$$

$$= \theta^{(\alpha+x)-1} e^{-(\beta+1)\theta}$$

$$\propto_{\theta} \text{Gamma}(\theta; \alpha+x, \beta+1)$$