

$(N_t)_{t \geq 0}$ POISSON PROCESS PARAM. λ

DEFINE $M_t = N_{t+s} - N_s$ FOR FIXED s , ALL $t \geq 0$

THEN $(M_t)_{t \geq 0}$ IS A POISSON PROCESS WITH PARAM. λ .

PROOF

COUNTING PROCESS

$$M_0 = 0$$

$$M_t \sim \text{POISSON}(\lambda t)$$

STATIONARITY, INDEP. INCREMENTS.

DEFINE X AS THE TIME OF THE FIRST EVENT. FOR ANY $t > 0$

$$P(X > t) = P(N_t = 0) = e^{-\lambda t} \quad \underline{P(X \leq t) = 1 - e^{-\lambda t}} \quad X \sim \text{EXPONENTIAL}(\lambda)$$