

$$E(G_0 e^{t\mu + \sigma B_t}) = G_0 e^{t\mu} \underline{E(e^{\sigma B_t})}$$

$$\underline{E(e^{\sigma B_t})} = \int_{-\infty}^{\infty} e^{\sigma s} \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{1}{2t} s^2\right) ds$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{1}{2t} s^2 + \sigma s\right) ds = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{1}{2t} (s - t\sigma)^2 + \frac{t\sigma^2}{2}\right) ds$$

$$= e^{t\sigma^2/2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{1}{2t} (s - t\sigma)^2\right) ds = \frac{e^{t\sigma^2/2}}{\underline{E(G_0 e^{t\mu + \sigma B_t})}} = G_0 e^{t\mu + t\sigma^2/2}$$

$$\text{Var}(G_0 e^{t\mu + \sigma B_t}) = G_0^2 e^{2t\mu} \cancel{E(e^{2\sigma B_t})} \text{Var}(e^{\sigma B_t}) = G_0^2 e^{2t\mu} (E(e^{2\sigma B_t}) - E(e^{\sigma B_t})^2)$$

$$= G_0^2 e^{2t\mu}$$