4.10

$1\,yr \simeq 3.2 \times 10^7 s$

$1\,ly = c \times 1\,yr = 3 \times 10^8 \frac{m}{s} \times 3.2 \cdot 10^7 s =$

$= 9.6 \times 10^{15} m.$

a): $g = 9.8 \frac{m}{s^2} = 9.8 \times \dfrac{\frac{1}{9.6} \times 10^{-15}\, ly}{\left(\frac{1}{3.2}\right)^2 \times 10^{-14}\, yr^2} =$

$= 9.8 \dfrac{(3.2)^2}{9.6} \times 10^{-1}\, ly/_{yr^2} \simeq 1.\, ly/yr^2.$

b). Set $c=1$ and use units ly, yr. Set also $\alpha = 1$ from a), so we can write in "dimensionless" units:

$$x = \frac{c^2}{\alpha}\left(\sqrt{1+\frac{\alpha^2 t^2}{c^2}} - 1\right) \Rightarrow x = \sqrt{1+t^2} - 1$$

$$v = \frac{\alpha t}{\sqrt{1+\frac{\alpha^2 t^2}{c^2}}} \Rightarrow v = \frac{t}{\sqrt{1+t^2}}$$

we know that $\gamma = \dfrac{1}{\sqrt{1-v^2}} = \dfrac{1}{\sqrt{1-\left(t/\sqrt{1+t^2}\right)^2}}$

$$= \frac{1}{\sqrt{\frac{1}{1+t^2}}} = \sqrt{1+t^2}$$

But we want to know γ as a funct. of τ

So must compute $t(\tau) = ?$

To do that, consider

$$d\tau^2 = dt^2 - dx^2$$

$$\Rightarrow \quad 1 = \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 =$$

$$= \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dt}{d\tau}\right)^2 \cdot \left(\frac{dx}{dt}\right)^2 =$$

$$= \left(\frac{dt}{d\tau}\right)^2 (1 - v^2) =$$

$$= \left(\frac{dt}{d\tau}\right)^2 \left(1 - \frac{t^2}{1+t^2}\right) =$$

$$= \left(\frac{dt}{d\tau}\right)^2 \frac{1}{1+t^2}$$

$$\Rightarrow \begin{cases} \dfrac{dt}{d\tau} = \sqrt{1+t^2} \\ t(0) = 0 \end{cases} \Rightarrow t = sh(\tau)$$

Put back into γ:

$$\gamma = \sqrt{1+t^2} = \sqrt{1+sh^2\tau} = ch\tau.$$

Also note: $x = \sqrt{1+t^2} - 1 = ch\tau - 1.$

	$\tau = 1$ day	1 yr	10 yr
$\gamma =$	1,000004	1,5	≈ 11000
$x =$	0,000004 ly	0,5 ly	≈ 11000 ly
$t =$	0,0027 yr	1,18 yr	≈ 11000 yr

c) The whole trip takes 4×10yr for the crew, but 4×11000 yr for us
the star is about 11000 ly away.