7,14

The energy of a CMBR photon is roughly (they are distributed with a black body distribution).

$$E_{CMBR} \simeq k_B T \simeq 2{,}3 \times 10^{-4}\ \text{eV}.$$

Assume the other photon has energy E_γ.

In a "head on" collision:

$$\underset{p^\mu_\gamma}{(E_\gamma, E_\gamma, 0\,0)} + \underset{p^\mu_{CMBR}}{(E_{CMBR}, -E_{CMBR}, 0\,0)}$$

$=$

$e^+ : p^\mu_{e^+}$

e^-

$p^\mu_{e^-}$

So: $P_\gamma^\mu + P_{CMBR}^\mu = P_{e^-}^\mu + P_{e^+}^\mu$

We can compare their squares in any ref. system since they are Lorentz invariant:

$$(E_\gamma + E_{CMBR})^2 - (E_\gamma - E_{CMBR})^2 = (2 m_e)^2$$

$$4 E_\gamma E_{CMBR} = 4 m_e^2$$

$$\Rightarrow \quad E_\gamma = \frac{m_e^2}{E_{CMBR}} = \frac{(0{,}511)^2 \text{ MeV}^2}{2{,}4 \times 10^{-4} \times 10^{-6} \text{ MeV}}$$

$$\simeq 1{,}1 \times 10^9 \text{ MeV} \quad (!).$$

A similar bound applies to protons (read about the GZK bound).