7.22

$\epsilon = h\nu \simeq 10^{-3}\,eV = 10^{-12}\,GeV$

BEFORE : Proton   CMPR photon

$P_p^\mu = (E, P; 0, 0)$ $P_\gamma^\mu = (\epsilon, -\epsilon, 0, 0)$

AFTER :

$q_\gamma^\mu = (\epsilon', \epsilon', 0\,0)$.

q_p^μ (not needed)

$$P_p^\mu + P_\gamma^\mu = q_\gamma^\mu + q_p^\mu$$

$$(P_p^\mu + P_\gamma^\mu - q_\gamma^\mu)^2 = (q_p^\mu)^2$$

$$P_p^2 + P_\gamma^2 + q_\gamma^2 + 2P_p \cdot P_\gamma - 2P_p \cdot q_\gamma - 2P_\gamma \cdot q_\gamma = q_p^2$$

$$m^2 + 0 + 0 + 2(E+P)\epsilon - 2(E-P)\epsilon' - 4\epsilon\epsilon' = m^2$$

// Note: $P_p \cdot P_\gamma = E\epsilon - P(-\epsilon) = (E+P)\epsilon$

$P_p \cdot q_\gamma = E\epsilon' - P\epsilon' = (E-P)\epsilon'$

$P_\gamma \cdot q_\gamma = \epsilon\epsilon' - (-\epsilon)\epsilon' = 2\epsilon\epsilon'$ //

Solve for ϵ':

$$\epsilon' = \frac{(E+P)\epsilon}{2\epsilon + E - P}$$

Now: $E^2 = p^2 + m^2 \Rightarrow p = \sqrt{E^2 - m^2}$

$= (\text{Taylor})\ E - \frac{m^2}{2E}$ ($m \simeq 1\,\text{GeV}$ proton mass.)

$$\Rightarrow \epsilon' \simeq \frac{2E\epsilon}{2\epsilon + \frac{m^2}{2E}} =$$

$$= \frac{2\times 10^{11}\,\text{GeV} \times 10^{-12}\,\text{GeV}}{2\times 10^{-12}\,\text{GeV} + \frac{1\,\text{GeV}^{\cancel{2}}}{2\times 10^{11}\,\cancel{\text{GeV}}}} =$$

$$= \frac{2\times 10^{-1}\,\text{GeV}}{2\times 10^{-12} + 5\times 10^{-12}} \sim 0{,}3\times 10^{11}\,\text{GeV}.$$

$$\sim 3\times 10^{19}\,\text{eV}.$$