Advanced Algorithms. Assignment 3

Exercise 5.
A random bit is a probability space with 2 elementary events, each appearing with probability 1/2 (in simple words: flipping a coin). Suppose that random bits are available, but we want to
(a) simulate an event with probability 1/3,
(b) sample from a probability space with 3 elementary events, each appearing with probability 1/3.

5.1. Can you solve (a) and (b) using some *guaranteed* finite number of random bits? Motivate your answers, that is, explain why you think this is possible or impossible.

5.2. Solve (a) and (b), respectively, by a randomized algorithm using some *expected* finite number of random bits. Try to make this number as small as you can. Of course, do not only describe your proposed algorithm, but also prove that it behaves as claimed.

5.3. (optional) You may add other results and reflections here, e.g., prove that your expected numbers in 5.2 are optimal, generalize the exercise to other probability values, or whatever you find interesting or remarkable,

Exercise 6.
Remember the problem from Exercise 1: Delete k nodes from a graph, such that, in the remaining graph, every connected component contains at most c nodes.
But this time we want an "exact" solution with at most k nodes (not a larger approximate solution), provided that such a small solution exists.
Describe an algorithm that solves this problem in $O^*((c+1)^k)$ time or better, and argue why your proposed algorithm is correct.