MATHEMATICS Univ.of Gothenburg and Chalmers University of Technology
Examination in algebra: MMG500 and MVE 150, 2019-06-10.
No books, written notes or any other aids are allowed.
Telephone 031-772 5325.

1. Which or the following statements are correct?
   a) Every integral domain of characteristic $p>0$ is a field. 1p
   b) Every field is an integral domain. 1p
   c) Every finite commutative ring is an integral domain. 1p

2. Let H be a subgroup of a group G and m,n be two integers with $(m,n)=1$. 4p
Let $g\in G$ be an element such that $g^m\in H$ and $g^n\in H$. Prove that $g\in H$.

3. Let $I_1\subseteq I_2 \subseteq \ldots \subseteq I_k\subseteq\ldots$ be an infinite chain of (possibly equal) ideals in 4p
$\mathbf{C}[x]$. Prove that this chain becomes stationary $I_n= I_{n+1}= \ldots$ after some $n\in \mathrm{N}$.

4 The sides of a cube are coloured in blue, green and red and two coloured 5p
cubes are identified if they are related by a rotational symmetry. Show that
there are exactly 57 such cubes.
(Only solutions based on group theory will receive points.)

5. Let $* : G\times G \to G$ be an associative binary operation on a set G. 4p
a) Show that $(G, *)$ has at most one neutral element.
b) Show that each element of G has at most one inverse with respect to $*$.

6. Formulate and prove Lagrange's theorem. 5p
(The proof should be complete and not based on any unproved lemmas except that you
may use the fact that the equivalence classes of an equivalence relation form a partition.)

The theorems in Durbin's book may be used to solve the first four exercises. But all claims that are made should be <u>motivated</u>.