MATHEMATICS
University of Gothenburg and Chalmers University of Technology
Examination in algebra : MMG 500 and MVE 150, 2017-06-08.
No books, written notes or any other aids are allowed.
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1a) Compute the product $\pi=(1\ 2)(2\ 3)(3\ 4)$ in S_4.   4p

b) Describe the permutations in the cyclic subgroup generated by π.

The permutations should be written in cycle form.

2 Let g,h be two elements in a finite group. Show that gh and hg have   4p

the same order.

3.Determine the zero divisors and invertible elements in $\mathbf{Z}_{10}$.   4p

4 Let p be a prime.   5p
a) Show that the equation $x^p -1 =0$ has no other root than 1 in $\mathbf{Z}_p$.
b) Can the equation $x^p - a =0$ have more than one root in $\mathbf{Z}_p$ for
other elements $a\neq 1$ in $\mathbf{Z}_p$?

5. Let $* : G\times G \rightarrow G$ be an associative binary operation on a set G.   4p

a) Show that $(G, *)$ has at most one neutral element.

b) Show that each element of G has at most one inverse with respect to $*$.

6. Show that any finite integral domain is a field.   4p

All claims that are made must be motivated.