MATHEMATICS
Univ. of Gothenburg and Chalmers University of Technology
Examination in algebra : MMG500 and MVE 150, 2019-08-21.
No aids are allowed. Telephone 031-772 5325.

1. Let R be the quotient ring $\mathbf{Z}_2[x]/(x^2+1)$. Write down the Cayley tables for addition and multiplication on R. 4p

(All cosets should be represented by binary polynomials of minimal degree.)

2. Find the subgroups of $\mathbf{Z}_2\times\mathbf{Z}_4$. (There are eight.) 4p

3. Let G be the abelian group of all rotations of the unit circle $S^1=\{(\cos\theta, \sin\theta)\in\mathbf{R}^2 : 0\le\theta<2\pi\}$. Determine the number of elements of order one million in G. (Hint: Prove first that $G\approx\mathbf{R}/2\pi\mathbf{Z}$.) 4p

4. Let $\varepsilon=\cos(2\pi/3)+i\sin(2\pi/3)=(-1+i\sqrt{3})/2$ and D be the set of all complex numbers of the form $a+b\varepsilon$ with $a,b\in\mathbf{Z}$

a) Show that D is a subring of $\mathbf{C}$. 2p

b) Show that the integral domain D is Euclidean by means of the function $\delta(a+b\varepsilon)=|a+b\varepsilon|^2 = a^2-ab+b^2$ 3p

5. Formulate and prove the fundamental homomorphism theorem for groups. 4p

6. Show that the kernel of a ring homomorphism $\theta: R\to S$ is an ideal of R by verifying <u>all</u> conditions for a subset of R to be an ideal. 4p

The theorems in Durbin's book may be used to solve exercises 1–4, *but all claims that are made must be motivated.*