

Excercise section 1: Repetition of basic statistics

MVE220/MSA400

Problems for exercise session 1

During the exercise session 27.03.2023 we will consider the following problems (translated from [1]))

- 1) Two dice are thrown. Assume their sum was 4. What is the conditional probability that
 - a) The first dice showed 3
 - b) The second dice showed 2 or less
 - c) Both dice showed an odd number
- 2) Let Y be a random variable with the cdf

$$F(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq t \leq 1 \\ 1 & t > 1 \end{cases}$$

- a) Sketch $F(t)$
 - b) Calculate $P(Y \leq 0.5)$
 - c) Calculate $P(0.5 < Y \leq 0.9)$
- 3) Calculate the expectation and standard deviation of X .
- 4) Let (X, Y) be given with pdf $f_{X,Y}(x, y) = x + y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. Calculate $E[X]$, $E[Y]$, $Var(X)$, $Var(Y)$, $Cov(X, Y)$ and $\rho(X, Y)$.
- 5) Let $\{N(t), t \geq 0\}$ be a Poisson process with $\lambda = 2$. Calculate
 - a) $P(N(1) = 0)$
 - b) $P(N(3) = 4)$
 - c) $P(N(2) \leq 3)$
 - d) $P(N(0.5) > 1)$
- 6) Nine observations were obtained from a distribution with pdf

$$f(x) = \frac{x}{\theta^2} e^{x/\theta}$$

Find the ML-estimate of θ .

k	1	2	3	4	5
$p_X(k)$	0.2	0.1	0.3	0.1	0.3

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
2.2	5.5	1.7	1.3	3.5	3.2	0.6	3.8	1.9

- 7) The random variable X is Poisson distributed with expected value μ . The 95% confidence interval for μ is $I_\mu = (0.8, 2.0)$. Calculate the 95% confidence interval for $p := P(X = 0)$.

Suggested problems that cover specific topics:

For further practice, I recommended the following problems from Introduction to Probability and Statistics: Principles and Applications for Engineering and the Computing Sciences[2].

- (i) Probability densities (chapter 3.2, 4.1): 3.8, 4.1, 4.9
- (ii) Independent events, conditional probabilities (chapter 2.2, 2.3): 2.15, 2.18
- (iii) Poisson distribution (chapter 3.8): 3.8, 3.61, 3.63
- (iv) Expected value, variance, moments (chapter 3.3): 3.13, 3.21
- (v) Correlation (chapter 5.3): 5.20, 5.27
- (vi) Point estimation (chapter 7.1): 7.5, 7.32
- (vii) Confidence intervals (chapter 7.4): 7.47

References

- [1] Sven Erick Alm and Tom Britton. *Stokastik: sannolikhetsteori och statistikteori med tillämpningar*. Liber, 2008.
- [2] Milton J Susan and Jesse C Arnold. *Introduction to Probability and Statistics: Principles and Applications for Engineering and the Computing Sciences*. John Wiley & Sons, 2003.