Exercises in Financial Risk: Extreme Value Statistics

MVE220/MSA400

1. In a peaks over thresholds analysis to compute VaR for the daily losses of a stock, the threshold $u = 2.2$ was chosen such that 3% of the losses were above u, and one then fitted a GP distribution to the excesses over the threshold. The estimated scale parameter was $\sigma = 0.71$ and the estimated shape parameter was $\gamma = 0.15$. What is then the estimated 99% daily VaR for the stock?

2. The block maxima method was applied to a data set and yielded the maximum likelihood estimate
$$(\hat{\mu}, \hat{\sigma}, \hat{\xi}) = (-1.64, 0.27, -0.084)$$
with a maximized value of the log-likelihood equal to -14.3. The corresponding estimated variance-covariance matrix is
$$V = \begin{bmatrix} 0.00141 & 0.000214 & -0.000795 \\ 0.000214 & 0.000652 & -0.000441 \\ -0.000795 & -0.000441 & 0.00489 \end{bmatrix}.$$
Compute a 95% confidence interval for the shape parameter ξ.

3. Suppose that the monthly maximum losses of a financial instrument has a GEV distribution with location parameter μ, scale parameter σ and shape parameter $\gamma \neq 0$, and that the monthly losses are independent. Then also the yearly maximum losses have a GEV distribution. What are the parameters of this distribution?

4. Suppose $X_1, X_2, ...$ are independent and exponentially distributed random variables with parameter σ (so that their cumulative distribution function is $F(x) = 1-e^{-x/\sigma}$ for $x \geq 0$ and $F(x) = 0$ for $x < 0$), and define $M_n = \max\{X_1, ..., X_n\}$. Find norming values a_n, b_n such that
$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \to e^{-e^{-x}}, \qquad \text{as } n \to \infty.$$

5. Suppose a data set contains 7-day losses (= - returns) during the last 2 years for a portfolio, and that the mean and standard deviation of the losses are -0.002 and 0.03, respectively. If one assumes that losses are normally distributed and that SEK 1.7 million is invested in the portfolio, what is then the 7-day 99% VaR, in SEK, for the portfolio?

6. Suppose that daily portfolio losses are identically distributed and that 7-day maximum losses expressed in % have a GEV distribution function with estimated location parameter $\hat{\mu} = 0.96$, estimated scale parameter $\hat{\sigma} = 1.17$, and estimated shape parameter $\hat{\gamma} = 0.21$, and that the extremal index has been estimated to be $\hat{\theta} = 0.73$. Compute an estimate for the 1-day 95% VaR for such a portfolio with an initial value of SEK 1.7 million.

7. Suppose that an excesses of a threshold u has a GP distribution with scale parameter σ and shape parameter γ, where it for simplicity is assumed that $\gamma > 0$. Assume that $v > 0$. Then the conditional distribution of excesses of the level $u+v$ is also a GP distribution. What are the parameters of this distribution?

8. In a peaks over thresholds analysis to compute VaR one assumed that from very long experience it was known that the 95% quantile of the losses was exactly 2.60, with negligible error. Further, one used a GP distribution with shape parameter $\gamma = 0$, so that the distribution function was $H(x) = 1 - e^{-x/\sigma}$, to model the excesses over the threshold 2.60. The estimated parameter was $\hat{\sigma} = 0.90$, and the standard deviation of $\hat{\sigma}$ was estimated to be 0.057. Let $\widehat{VaR}_{.8}(H)$ be the the estimated 80% VaR of the distribution function H, so that the estimated 0.99 VaR of the losses equals $2.60 + \widehat{VaR}_{.8}(H)$. Find a 95% confidence interval for the 99% VaR of the losses.

9. Suppose that 5-day maximum losses are mutually independent and have a GEV distribution function with location parameter μ, scale parameter σ, and shape parameter γ. Then 20-day maximum losses also have a GEV distribution. What are the location, scale, and shape parameter for this distribution?

10. Suppose that the 5-day maximum loss (= - returns) has a GEV distribution function with location parameter $\mu = 1.27$, scale parameter $\sigma = 0.79$ and shape parameter $\gamma = 0.14$, and that additionally the extremal index is $\theta = 0.73$. Compute the one-day VaR from this.

11. In a peaks over thresholds analysis to compute VaR one used the threshold $u = 1.7$ chosen such that 5% of the losses were above u and fitted a GP distribution to the excesses over the thresholds. The estimated parameters were $\hat{\sigma} = 0.8$ and $\hat{\gamma} = 0.1$. Find an estimate of the 99% VaR.

12. Suppose $X_1, X_2, ...$ are independent and uniformly distributed random variables (so that their cumulative distribution function is $F(x) = x$ for $x \in [0; 1]$), and define $M_n = \max\{X_1, ..., X_n\}$. Show that for normalizing constants $a_n = 1, b_n = \frac{1}{n}$ it holds that

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \to e^x, \qquad \text{for } x \in [0, 1], \text{ as } n \to \infty.$$

13. Let X_1 and X_2 be independent random variables which both have a GEV distribution function with shape parameter $\gamma = 0$, scale parameter $\sigma = 1.3$ and location parameter $\mu = 2.1$, so that they both have cumulative distribution function

$$G(x) = e^{-e^{-\frac{x-2.1}{1.3}}}.$$

Then also $M = \max\{X_1, X_2\}$, the maximum of X_1 and X_2, has a GEV distribution function. Find the shape, scale, and location parameters of this GEV distribution function of M.

14. The block maxima method was applied to a data set on glass fibre strengths and yielded the maximum likelihood estimate

$$(\hat{\mu}, \hat{\sigma}, \hat{\xi}) = (-1.64, 0.27, -0.084)$$

with a maximized value of the log-likelihood equal to -14.3. The corresponding estimated variance-covariance matrix is

$$V = \begin{bmatrix} 0.00141 & 0.000214 & -0.000795 \\ 0.000214 & 0.000652 & -0.000441 \\ -0.000795 & -0.000441 & 0.00489 \end{bmatrix}.$$

Compute a 95% confidence interval for the scale parameter σ.

15. Suppose you have observed a sample $x_1, x_2, ..., x_n$ from a GP distribution with shape parameter $\gamma = 0$ and scale parameter σ.

a) Find the maximum likelihood estimator $\hat{\sigma}$ of σ.

b) Use observed information to find an estimate of the variance of $\hat{\sigma}$.

16. For the PoT model with threshold u, suppose that $P(L > u) = p_u$. Show that for $p > p_u$,

$$VaR_p(L) = \frac{\sigma}{\gamma}\left\{\left(\frac{1-p}{p_u}\right)^{-\gamma} - 1\right\} + u$$

and

$$ES_p(L) = VaR_p(L) + \frac{\sigma + \gamma(VaR_p(L) - u}{1-\gamma}$$

17. A portfolio of loans may lead to the losses L with the probabilities in the table below. Calculate $ES_\alpha(L)$ for

a) $\alpha = 0.95$

b) $\alpha = 0.99$

loss (million dollars)	1	2	5	10	20	30	40
probability (%)	40	33	10	12	2	2.7	0.3

18. Loss L follows the exponential distribution $L \sim \exp(1)$, i.e. $F(x) = \begin{cases} 1 - e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$

Calculate $ES_{0.95}(L)$.