

1 Matlab codes for illustrations.

We put here Matlab codes used for drawing phase portraits in the Introduction and for examples of linear autonomous systems in the plane.

Pendulum.

```
t0 = 0; % starttime
tend = 20; % finish time
xlabel('\theta (t)');
ylabel('d \theta / dt (t)');
axis equal
axis([-3*pi,3*pi, -4, 4]);
hold on;
x1=linspace(-3*pi,3*pi ,40);
x2=linspace(-4,4 ,40);
[X1,X2]=meshgrid(x1,x2);
quiver(X1,X2,X2,-sin(X1));
pause;
for k=-2:2:4
    plot([k*pi,k*pi],[0,0], 'r*');
    plot([-pi+k*pi,-pi+k*pi],[0,0], 'b*'); %drawing equilibrium points
end

button=1; % index meaning that the left button on the mouse is pushed

while button==1 % condition checking that the left button is pushed
    [i,j,button]=ginput(1); % reading coordinates of the mouse index on the screen
    chosen by the left button
    [~, y] = ode45(@(t,y)[ y(2); -sin(y(1))], [t0 tend], [i; j]); % start angle i, start
    angle velocity j
    plot(y(:,1),y(:,2), 'b');
end
```

Van der Pol equation

```
t0 = 0; % starttid
tend = 20; % sluttid
A=[0 , 1; 1 , 0];
[V,D]=eig(A)
```

```

button=1;
xlabel('x1');
ylabel('x2');
axis equal
axis([-5,5, -5, 5])
title('Phase plane for Van der Pol equation' );
hold on;
g=1;
plot([0,0],[0,0], 'r*');
while button==1; % checking that the left button is pushed
[i,j,button]=ginput(1);
[~, y] = ode45(@(t,y)[y(2);-y(1)+g*y(2)*(1-y(2)*y(2))], [t0 tend], [i; j]); %
startvinkel i, starthastighet j

```

```

if button==1 plot(y(:,1),y(:,2), 'b'); % checking that the left button is pushed
else plot(y(:,1),y(:,2), 'r','LineWidth',3); %the last orbit will be red
end
end
hold of

```

Linear autonomous system in plane

```

t0 = 0; % starttid
tend = 20; % sluttid
A=[0 , 1; 1 , 0];
[V,D]=eig(A)
button=1;
xlabel('x1');
ylabel('x2');
axis equal
axis([-5,5, -5, 5])

hold on;
L=10;
plot([-L*V(1,1);L*V(1,1)],[-L*V(2,1);L*V(2,1)], 'r');
plot([-L*V(1,2);L*V(1,2)],[-L*V(2,2);L*V(2,2)], 'r');
while button==1; % checking that the left button is pushed
[i,j,button]=ginput(1);
[~, y] = ode45(@(t,y)A*y, [t0 tend], [i; j]); % startvinkel i, starthastighet j

```

```
plot(y(:,1),y(:,2), 'b');  
end
```