Formelblad

- **Diracs deltafunktion**:

$$f(a) = \int_{-\infty}^{\infty} \delta(x-a) f(x)\, dx, \qquad \delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dx \tag{1}$$

- **Stegoperatorer** för en harmonisk oscillator, $V(\hat{x}) = \frac{1}{2}k\hat{x}^2 = \frac{1}{2}m\omega_0^2\hat{x}^2$:

$$\hat{a}^\dagger = \frac{\hat{x}}{2L} - \frac{iL\hat{p}}{\hbar}, \qquad \hat{a} = \frac{\hat{x}}{2L} + \frac{iL\hat{p}}{\hbar} \tag{2}$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad \hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle \tag{3}$$

  där $L = \sqrt{\hbar/(2m\omega_0)}$.

- **Paulimatriser** för j eller $s = 1/2$:
  $\hat{J}_i|\hat{S}_i = \frac{\hbar}{2}\sigma_i$.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

- **Paulimatriser** för j, ℓ eller $s = 1$:
  $\hat{J}_i|\hat{L}_i|\hat{S}_i = \hbar\sigma_i$.

$$\sigma_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \sigma_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

- **Clebsch-Gordan tabeller**
  Kvadrot antagen för varje koefficient. Minustecken ska stå framför roten.

$s_1 = \frac{1}{2}$		1			
$s_2 = \frac{1}{2}$		1	1	0	
1/2	1/2	1	0	0	
	1/2	−1/2	1/2	1/2	1
	−1/2	1/2	1/2	−1/2	−1
			−1/2	−1/2	1

$s_1 = 1$		3/2					
$s_2 = \frac{1}{2}$		3/2	3/2	1/2			
1	1/2	1	1/2	1/2			
	1	−1/2	1/3	2/3	3/2	1/2	
	0	1/2	2/3	−1/3	−1/2	−1/2	
			0	−1/2	2/3	1/3	3/2
			−1	1/2	1/3	−2/3	−3/2
					−1	−1/2	1

- Hamiltonianen i **sfäriska koordinater**:

$$\hat{H} = \frac{\hat{p}_r^2}{2m} + \frac{\hat{L}^2}{2mr^2} + V(\mathbf{r}), \tag{6}$$

där

$$\hat{p}_r^2 = \left[-i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\right]^2 = -\frac{\hbar^2}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) \tag{7}$$

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \tag{8}$$