

$$\pi(p|x) \propto_p \pi(x|p) \pi(p)$$

$$\propto_p p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k} p_1^{\alpha_1-1} p_2^{\alpha_2-1} \cdots p_k^{\alpha_k-1}$$

$$= p_1^{\alpha_1+x_1-1} \cdots p_k^{\alpha_k+x_k-1}$$

$$\propto_p \text{DIRICHLET}(p; \alpha + x)$$

$$p \sim \text{DIRICHLET}(2, 4, 1)$$

$$E(p) = \frac{(2, 4, 1)}{7} = \left(\frac{2}{7}, \frac{4}{7}, \frac{1}{7}\right)$$