

$$\begin{aligned}\pi(x) &= \frac{\pi(x|r)\pi(r)}{\pi(r|x)} \\ &= \frac{\text{MULTINOMIAL}(x; r) \text{DIRICHLET}(r; \alpha)}{\text{DIRICHLET}(r; \alpha + x)}\end{aligned}$$

$$\begin{aligned}&= \\ \pi(x = e_i) &= \frac{\Gamma(\alpha_i + 1)}{\Gamma(\alpha_i)} \cdot \frac{\Gamma(\sum \alpha_j)}{\Gamma(1 + \sum \alpha_j)} \\ &= \frac{\alpha_i}{\sum_j \alpha_j}\end{aligned}$$

$$\boxed{\Gamma(x+1) = x \Gamma(x)}$$