ASSUME $X_i \in \{1, 2, 3\}$ $X_0 = 1$

Y_i: NON-NEGATIVE INTEGERS

$Y_i | X_i \sim \text{POISSON}(\lambda X_i)$

PRIOR FOR λ: $\pi(\lambda) \propto \frac{1}{\lambda}$

PRIOR FOR P:

$P_1 \sim \text{DIRICHLET}(1, 1, 0)$

$P_2 \sim \text{DIRICHLET}(1, 1, 1)$

$P_3 \sim \text{DIRICHLET}(0, 1, 1)$

OBSERVED DATA:

X	1	2	2	3	3	2	1	1	(sum 15)
Y	4	5	7	10	11	8	5	3	(sum 53)

POSTERIOR:

$P_1 | \text{data} \sim \text{DIRICHLET}(2, 2, 0)$

$P_2 | \text{data} \sim \text{DIRICHLET}(2, 2, 2)$

$P_3 | \text{data} \sim \text{DIRICHLET}(0, 2, 2)$

(TRANS. $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$)

$Y | \lambda X \sim \text{POISSON}(\lambda X)$

$\pi_i(y) = e^{-\lambda x} \frac{(\lambda x)^y}{y!}$

POSTERIOR FOR λ:

$\pi(\lambda | \text{data}) \propto_\lambda \pi(\text{data} | \lambda) \pi(\lambda)$

$\propto_\lambda e^{-\lambda} (\lambda)^4 \cdot e^{-2\lambda} (2\lambda)^5 \cdot \dots \cdot \frac{1}{\lambda}$

$\propto e^{-15\lambda} \lambda^{57-1} \propto \text{GAMMA}(\lambda; 57, 15)$