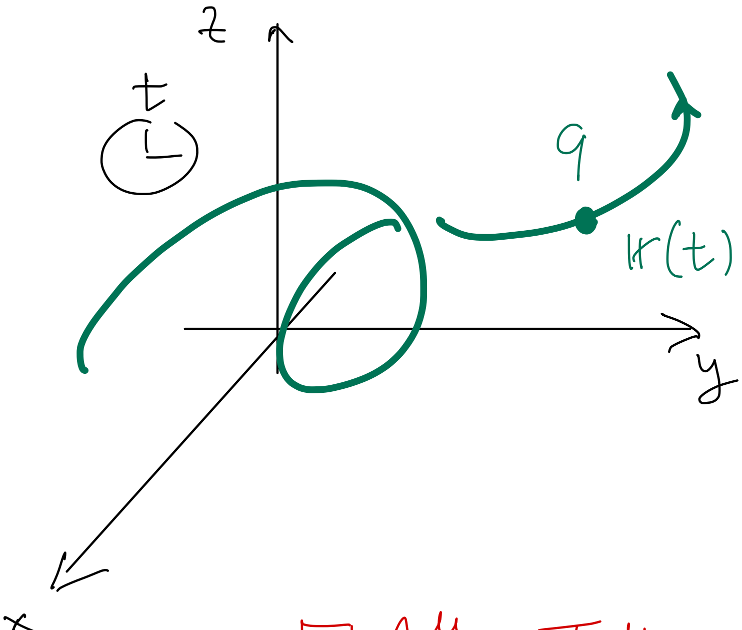
SPECIELL RELATIVITETSTEORI 4

Strålning från rörlig laddning (LIENARD–WIECHERT POTENTIAL)



$\mathbf{r}(t)$ (känd).

$$\rho(t,\mathbf{x}) = q\,\delta(\mathbf{x}-\mathbf{r}(t))$$

$$\mathbf{J}(t,\mathbf{x}) = q\,\dot{\mathbf{r}}(t)\,\delta(\mathbf{x}-\mathbf{r}(t))$$

$$J^\mu = (\rho, \mathbf{J})$$

$$\partial_\mu J^\mu = 0$$

$\Box A^\mu = J^\mu$ uppfyller $\partial_\mu A^\mu = 0$

OK.

GREENSFUNKTION!

$$\Box G(t,\mathbb{x}) = \delta(t)\,\delta^{(3)}(\mathbb{x})$$

$$G(t,\mathbb{x}) = \delta(t-r)\cdot\frac{1}{4\pi r} \qquad (r = |\mathbb{x}|.)$$

$$\Rightarrow A^\mu(t,\mathbb{x}) = \int d^3y\,dt'\; G(t-t',\mathbb{x}-\mathbb{y})\cdot J^\mu(t',\mathbb{y})$$

eg $\mu = 0$:

$$V(t,x) = \int d^3y\,dt'\;\delta(t-t'-|\mathbb{x}-\mathbb{y}|)\,\frac{1}{4\pi|\mathbb{x}-\mathbb{y}|}\,q\,\delta(\mathbb{y}-\mathbb{r}(t'))$$

$$V(t,\mathbf{x}) = \int d^3y\, dt' \, \delta(t-t'-|\mathbf{x}-\mathbf{y}|)\frac{1}{4\pi|\mathbf{x}-\mathbf{y}|} q\,\delta(\mathbf{y}-\mathbf{r}(t'))$$

$$= \int dt' \, \delta(t-t'-|\mathbf{x}-\mathbf{r}(t')|)\cdot\frac{q}{4\pi|\mathbf{x}-\mathbf{r}(t')|} =$$

$$\frac{q}{4\pi|\mathbf{x}-\mathbf{r}(\tilde{t})|}\cdot\frac{1}{\left|\frac{d}{dt'}\left(t-t'-|\mathbf{x}-\mathbf{r}(t')|\right)\right|} = \quad \left(\tilde{t}:\ t-\tilde{t} = |\mathbf{x}-\mathbf{r}(\tilde{t})|\right)$$

$$\frac{q}{4\pi|\mathbf{x}-\mathbf{r}(\tilde{t})|}\cdot\frac{1}{1+\frac{(\mathbf{r}(\tilde{t})-\mathbf{x})\cdot\dot{\mathbf{r}}(\tilde{t})}{|\mathbf{x}-\mathbf{r}(\tilde{t})|}} = \frac{q}{4\pi\left(|\mathbf{x}-\mathbf{r}(\tilde{t})|+(\mathbf{r}(\tilde{t})-\mathbf{x})\cdot\dot{\mathbf{r}}(\tilde{t})\right)}$$

$$V(t,x) = \frac{q}{4\pi\left(|x - r(\tilde{t})| + (r(\tilde{t}) - x)\cdot \dot{r}(\tilde{t})\right)}$$

$$A(t,x) = \frac{q\,\dot{r}(\tilde{t})}{4\pi\left(|x - r(\tilde{t})| + (r(\tilde{t}) - x)\cdot \dot{r}(\tilde{t})\right)}$$

Fälten i t, x beror på laddning i

$r(\tilde{t})$ $\quad t = \tilde{t} + |x - r(\tilde{t})|$

MINSTA VERKANS PRINCIP.

Vi kan härleda $\partial_\mu F^{\mu\nu} = J^\nu$ från:

$$S = \int \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\nu J^\nu \right) d^4x .$$

$\left(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \text{ från början.} \right)$.

Vi kan använda Euler Lagrange ekv men det enklaste sättet är att göra härledningen från början:

$$S = \int \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\nu J^\nu \right) d^4x \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$A_\mu \to A_\mu + \delta A_\mu \Rightarrow F_{\mu\nu} \to \partial_\mu (A_\nu + \delta A_\nu) - \partial_\nu (A_\mu + \delta A_\mu)$$

$$= F_{\mu\nu} + \underbrace{\partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu}_{\delta F_{\mu\nu}}.$$

$$\Rightarrow \delta (F_{\mu\nu} F^{\mu\nu}) = \delta F_{\mu\nu} F^{\mu\nu} + F_{\mu\nu} \delta F^{\mu\nu} = 2 \delta F_{\mu\nu} F^{\mu\nu} =$$

$$= 2 (\partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu) F^{\mu\nu} = 4 \partial_\mu \delta A_\nu F^{\mu\nu}$$

$$\Rightarrow \quad \delta S = \int \left(-\frac{1}{4}\, \delta\left(F_{\mu\nu} F^{\mu\nu}\right) - \delta A_\nu J^\nu \right) d^4x =$$

yttre källa
varieras ej

$$= \int \left(-\cancel{\tfrac{1}{4}}\, \cancel{4}\, \partial_\mu \delta A_\nu F^{\mu\nu} - \delta A_\nu J^\nu \right) d^4x =$$

$$= \int + \delta A_\nu \left(\partial_\mu F^{\mu\nu} - J^\nu \right) d^4x \qquad \forall \delta A_\nu$$

$$\Rightarrow \quad \partial_\mu F^{\mu\nu} = J^\nu .$$

STRESSENERGI TENSOR:

$$T^{\mu\nu} = -F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{1}{4}\eta^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

- SYMMETRISK : $T^{\mu\nu} = T^{\nu\mu}$
- "SPÅRLÖS" : $T^{\mu}{}_{\mu} = \eta_{\mu\nu}T^{\mu\nu} = 0$
- BEVARAD : $\partial_{\mu}T^{\mu\nu} = 0$

VISA ATT:

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\mathbb{E}^2 - \mathbb{B}^2\right)$$

(T − V i elektromagn.)

$$T^{00} = \frac{1}{2}\left(\mathbb{E}^2 + \mathbb{B}^2\right)$$

(Energi tätheten)

$$T^{0i} = \mathbb{E} \times \mathbb{B}$$

(Poynting vektor)