Homework 1
Modelling with and Solving LPs

1. Consider the following LP problem:

$$\begin{aligned} \max \quad & 4x_1 - 2x_2 + 5x_3 + 6x_4 + 7x_5 \\ \text{s.t.} \quad & 2x_1 + 2x_2 - 4x_3 + 4x_4 + 8x_5 \leq 6 \\ & 2x_1 + x_2 - 2x_3 - x_4 - 3x_5 \geq -1 \\ & 5x_1 - 2x_2 + 4x_3 + 4x_4 + 2x_5 = 5 \\ & 2x_1 - 2x_2 + 5x_3 + 3x_4 + x_5 \leq 4 \\ & \vec{x} \geq \vec{0} \end{aligned}$$

   Use CVXOPT to solve the LP above. Submit your code (as a Python script), and write down the solution vector and objective value.

2. There are 4 space colonies, each of which requires a certain number of plasma conduits. There are 3 starbases in the vicinity. Each of them has total number of conduits they can spare and supply to the colonies. For each pair of starbase and colony, there is an associated cost for sending a cargo ship (each of which carries one plasma conduit), as shown in the table below:

	Triacus	New Berlin	Strnad	Vega	supply
Farpoint	6	9	10	8	35
Yorktown	9	5	16	14	40
Earhart	12	7	13	9	50
demand	20	30	30	45	$\left(\sum = 125\right)$

   Your goal is to supply the colonies the plasma conduits they need, at minimum cost.

   (a) (5 points) Consider the general *transportation problem*: where there are n colonies and m bases and the costs are given by a $m \times n$ matrix C, demand and supply are given by arrays d and s respectively. Formulate a LP to solve the problem.

   (b) (3 points) Code the LP in CVX, input the data for the space colonies manually and use CVX to solve the LP. Submit your code and write down the solution and objective

   (c) (2 points) Use CVX to show what the effect on the model and the optimal solution would be if each of the starbases could supply five more conduits.

3. (Not to be turned in). Use CVX to model and solve the other examples in the textbook (sections 2.1 – 2.6).