Homework 3
LP Duality and the Primal-Dual Algorithm

1. Consider the LP problem:

$$\begin{aligned} \max \quad & 4x_1 - 2x_2 + 5x_3 + 6x_4 + 7x_5 \\ \text{s.t.} \quad & 2x_1 + 2x_2 - 4x_3 + 4x_4 + 8x_5 \leq 6 \\ & 2x_1 + x_2 - 2x_3 - x_4 - 3x_5 \geq -1 \\ & 5x_1 - 2x_2 + 4x_3 + 4x_4 + 2x_5 = 5 \\ & 2x_1 - 2x_2 + 5x_3 + 3x_4 + x_5 \leq 4 \\ & \vec{x} \geq \vec{0} \end{aligned}$$

   (a) (4 points) Write the LP dual of this problem.

   (b) (3 points) Use CVX to compute the primal and dual optimum solutions and compare their values.

   (c) (3 points) Check the complementary slackness conditions.

2. Consider the LP problem:

$$\begin{aligned} \max \quad & 6x_1 - 5x_3 \\ \text{s.t.} \quad & 6x_1 - 3x_2 + x_3 = 2 \\ & 3x_1 + 4x_2 + x_3 \leq 5 \\ & x_1 - 7x_2 \leq 5 \end{aligned}$$

$$x_1 \geq 0, x_2 \leq 0, x_3 \text{unrestricted}$$

   (a) (3 points) Write the LP dual of this problem.

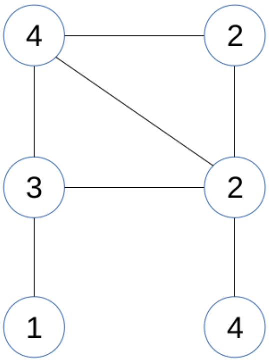
   (b) (4 points) Consider the feasible solution $\vec{x}^{:} = (0, 0, 2))$ to the primal. Check if this is optimal by using the complementary slackness conditions to write down the corresponding dual solution.

   (c) (3 points) Use complementary slackness to check if the primal feasible solution $\vec{x} = (1, 0, -4)$ is optimal.

3. Consider the primal-dual algorithm for vertex cover dicussed in class.

   (a) (4 points) Run it by hand on the graph in the figure below (from your previous homework). Show the values of the primal and dual variables at each iteration.

   (b) (6 points) Implement the primal–dual algorithm as a python script to compute (approximate) vertex covers and run it the random graph $G(200, 0.1)$ from the previous homework.


4
2
3
2
1
4