Chalmers University of Technology/Gothenburg University
Mathematical Sciences
Emil Gustavsson

TMA947 Nonlinear optimisation, 7.5 credits
MMG621 Nonlinear optimisation, 7.5 credits

The purpose of this basic course in optimization is to provide

(I) knowledge of some important classes of optimization problems and of application areas of optimization modelling and methods;

(II) practice in describing relevant parts of a real-world problem in a mathematical optimization model;

(III) an understanding of necessary and sufficient optimality criteria, of their consequences, and of the basic mathematical theory upon which they are built;

(IV) examples of optimization algorithms that are naturally developed from this theory, their convergence analysis, and their application to practical optimization problems.

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LECTURER/COURSE RESPONSIBLE: Emil Gustavsson, PhD, Business area leader within Machine Learning, Data Science, and Optimization, Fraunhofer-Chalmers Centre, http://www.fcc.chalmers.se, e-mail: emil.gustavsson@fcc.chalmers.se

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Course presentation

CONTENTS: The main focus of the course is on optimization problems in continuous variables; it builds a foundation for the analysis of an optimization problem. We can roughly separate the material into the following areas:

Convex analysis: convex set, polytope, polyhedron, cone, representation theorem, extreme point, Farkas Lemma, convex function

Optimality conditions and duality: global/local optimum, existence and uniqueness of optimal solutions, variational inequality, Karush–Kuhn–Tucker (KKT) conditions, complementarity conditions, Lagrange multiplier, Lagrangian dual problem, global optimality conditions, weak/strong duality

Linear programming (LP): LP models, LP algebra and geometry, basic feasible solution (BFS), the Simplex method, termination, LP duality, optimality conditions, strong duality, complementarity, interior point methods, sensitivity analysis

Nonlinear optimization methods: direction of descent, line search, (quasi-)Newton method, Frank–Wolfe method, gradient projection, exterior and interior penalty, sequential quadratic programming

We also touch upon other important problem areas within optimization, such as integer programming and network optimization.

PREREQUISITES: Passed courses on analysis (in one and several variables) and linear algebra; familiarity with matrix/vector notation and calculus, differential calculus. Reading Chapter 2 in the book (i) below provides a partial background, especially to the mathematical notation used and most of the important basic mathematical terminology.

ORGANIZATION: Lectures, exercises, a project assignment, and computer exercises.

COURSE LITERATURE:

(i) *An Introduction to Continuous Optimization, 3rd edition* by N. Andréasson, A. Evgrafov, E. Gustavsson, Z. Nedělková, M. Patriksson, K. C. Sou, and M. Önnheim, published by Studentlitteratur in 2016 and found in the Cremona book store

(ii) Hand-outs from books and articles

COURSE REQUIREMENTS: The course content is defined by the literature references in the plan below.

EXAMINATION:

- Written exam (first opportunity 31/10, 8.30–13.30)—gives 6 credits
- Project assignment—gives 1.5 credits
- Two correctly solved computer exercises

BONUS SYSTEM:

- Active participation during *exercises* gives at most 2 bonus points
- The bonus points are valid one year

COURSE EVALUATION: Three meetings between the Examiner and randomly selected course representatives will be organized. All students will be asked to fill a questionnaire.

SCHEDULE:

Lectures: on Mondays 08.00–09.45 and Tuesdays 15.15–17.00. Lectures are given in English. For locations, see the schedule below.

Exercises: on Mondays 10.00–11.45 and Fridays 08.00–09.45 in two parallel groups. For locations, see the schedule below.

Project: Teachers are available for questions in the computer rooms, which are also booked for work on the project, on 3/10 (room: MVF25) at 15.15–19.00. (Presence is not obligatory.) At other times, work is done individually. Deadline for handing in the project model (part 1): 27/9. Deadline for handing in the project report (part 2): 11/10.

Computer exercises: The computer exercises are scheduled to take place when also teachers are available, Computer exercise 1 on 19/9, 26/9 and Computer exercise 2 on 10/10 and 17/10 (room booked: MVF25), and on all occasions at 15.15–19.00. (Presence is not obligatory.) The computer exercises can be performed individually, but preferably in groups of two (and *strictly not* more than two). Deadline for handing in the report, unless passed through oral examination on site during the scheduled sessions: 3/10 (Computer exercise 1), 20/10 (Computer exercise 2).

Important note: The computer exercises *require* at least one hour of preparation each; having done that preparation, two–three hours should be enough to complete an exercise by the computer.

Information about the project and computer exercises are found on the web page
`http://www.math.chalmers.se/Math/Grundutb/CTH/tma947/1920/`

This course information, the course literature, project and computer exercise materials, most hand-outs and previous exams will also be found on this page.

COURSE PLAN, LECTURES:

Le 1 **(3/9)** *Course presentation.* Subject description. **Week 1**
Course map. Applications. Notations.
Optimization modelling. Modelling. Problem analysis. Classification.
(i): Chapter 1, 2

Le 2 **(9/9)** *Convexity.* Convex sets and functions. Polyhedra. The Representation Theorem. Fourier elimination. Farkas' Lemma. **Week 2**
(i): Chapter 3

Le 3 **(10/9)** *Optimality conditions, introduction.* Local and global optimality. Existence of optimal solutions. Feasible directions. Necessary and sufficient conditions for local or global optimality when the feasible set is convex. The Separation Theorem.
(i): Chapter 4

Le 4 **(16/9)** *Unconstrained optimization methods.* Search directions. Line searches. **Week 3**

Termination criteria. Steepest descent. Derivative-free methods.
(i): Chapter 11

<u>**Le 5**</u> **(17/9)** *Optimality conditions.* Introduction to the primal–dual optimality conditions. Geometric optimality conditions. The Fritz John optimality conditions.
(i): Chapter 5.1–5.4

<u>**Le 6**</u> **(23/9)** *The Karush–Kuhn–Tucker conditions.* Constraint qualifications. The Karush–Kuhn–Tucker conditions: necessary and sufficient conditions for local or global optimality. **Week 4**
(i): Chapter 5.5–5.9

<u>**Le 7**</u> **(24/9)** *Convex duality.* The Lagrangian dual problem. Weak and strong duality. Obtaining the primal solution.
(i): Chapter 6

<u>**Le 8**</u> **(30/9)** *Linear programming.* Introduction to linear programming. Modelling. Basic feasible solutions and extreme points (algebra versus geometry in linear programming). The simplex method, introduction. **Week 5**
(i): Chapter 7, 8

<u>**Le 9**</u> **(1/10)** *Linear programming, continued.* The Simplex method. The revised Simplex method. Phase I and II. Degeneracy. Termination. Complexity.
(i): Chapter 9

<u>**Le 10**</u> **(7/10)** *Linear programming duality.* Sensitivity analysis. **Week 6**
(i): Chapter 10

<u>**Le 11**</u> **(8/10)** *Convex optimization.* Optimality conditions over convex sets. Subgradient methods.
(i): Chapter 3, 4.4, 6.4

<u>**Le 12**</u> **(14/10)** *Integer programming.* Applications. Modelling. **Week 7**
(ii): On integer programming

<u>**Le 13**</u> **(15/10)** *Nonlinear optimization methods: convex feasible sets.* The gradient projection method. The Frank–Wolfe method. Simplicial decomposition. Applications.
(i): Chapter 12

Le 14 **(21/12)** *Nonlinear optimization methods: general sets.* Penalty and barrier methods. Interior point methods for linear programming, orientation. **Week 8**
(i): Chapter 13

Le 15 **(22/12)** *An overview of the course.*

COURSE PLAN, EXERCISES:

Ex 1 (6/9) Modelling. **Week 1**
(i): Chapter 1

Ex 2 (9/9) Convexity. Polyhedra. Representation. Farkas' Lemma. **Week 2**
(i): Chapter 3

Ex 3 (13/9) Local and global minimum. Feasible sets. Optimality conditions. Weierstrass' Theorem. Separation.
(i): Chapter 4

Ex 4 (16/9) Unconstrained optimization. **Week 3**
(i): Chapter 11

Ex 5 (20/9) The KKT conditions.
(i): Chapter 5

Ex 6 (23/9) Lagrangian duality. **Week 4**
(i): Chapter 6

Ex 7 (27/9) Geometric solution of LP problems. Standard form. The geometry of the Simplex method. Basic feasible solution.
(i): Chapters 7, 8

Ex 8 (30/9) The Revised Simplex method. Phase I & II.
(i): Chapter 9

Ex 9 (4/10) Duality in linear programming. **Week 5**
(i): Chapter 10.1–10.4

Ex 10 (7/10) Sensitivity analysis in linear programming.
(i): Chapter 10.5

Ex 11 (11/10) Subgradient optimization methods. **Week 6**
(i): Chapter 6.4

<u>**Ex 12**</u> **(14/10)** Algorithms for convexly constrained optimization. **Week 7**
The Frank–Wolfe and simplicial decomposition algorithms.
(i): Chapter 12

<u>**Ex 13**</u> **(18/10)** Constrained optimization methods. Penalty methods. Repetition.
(i): Chapter 13

<u>**Ex 14**</u> **(21/10)** Old exam. **Week 8**