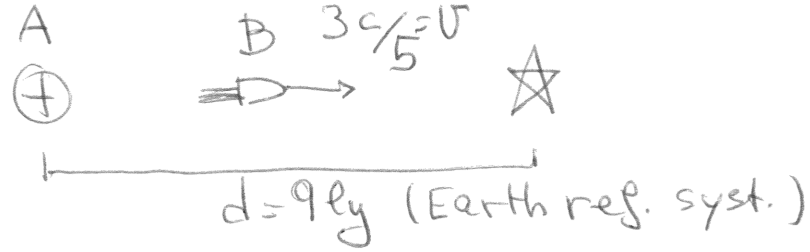
5.10



- We must use the Doppler formula because we are dealing with what an observer sees, so we must account for the extra time delay.

Inverting the formula for $\nu \sim \frac{1}{\Delta t}$:

$$\Delta t_A = \sqrt{\frac{1 - v/c}{1 + v/c}}\, \Delta t_B = \frac{1}{2} \Delta t_B \text{ from A's frame.}$$

So an observer A on Earth sees B move, age, .. half as fast.

THE SAME THING happens for B. She sees A move, age, .. half as fast on her way to the planet.

- B arrives on the planet after $d/v = 9 \cdot \frac{5}{3} = 15$ Earth years but A on Earth sees the arrival after $15 + 9 = 24$ yr (light must come back from the planet.)

- Under the whole 24 yr perod B has aged only $\frac{1}{2} \times 24 = 12$ yr.

- On the way back $v \to -v$:

$$\Delta t_A = \sqrt{\frac{1+v/c}{1-v/c}}\,\Delta t_B = 2\Delta t_B$$

Earth(A) sees B age twice as fast on the way back. THE SAME IS TRUE for B.

- The trip back looks on the monitor only $30-24 = 6\,yr$ long. (Alternatively $15-9 = 6\,yr$).
- B has aged $2 \times 6 = 12\,yr$ on the way back. All together, B has aged $12+12 = 24\,yr$ against $15+15 = 30\,yr$ of A.
- From B's point of view the planet is only $d' = d/\gamma = 9\cdot\sqrt{1-\left(\frac{3}{5}\right)^2} = \frac{36}{5}\,ly$
- The trip to the planet takes for B: $\frac{36}{5} \times \frac{5}{3} = 12\,yr$ (as before!)
- During the trip to the planet, B sees A age, move,.. half as fast. When B arrives, A looks $12 \times \frac{1}{2} = 6\,yr$ older

- The trip back takes B exactly the same amount : 12 yr
- During the trip back, B sees A age twice as fast : $12 \times 2 = 24$ yr

So all together A has aged

$6 + 24 = 30$ yr As before!