CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045/FYP302 - Speciell Relativitetsteori. 2018-01-09

Examinator: Gabriele Ferretti   rum: Soliden S3039
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OBS: Nästa granskningstillfälle: 2018-02-02, 16:00-17:00 i Origo N6115

Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook

Tentamen innehåller 6 uppgifter. Varje uppgift ger max 5 poäng.
Betygsgränser: CTH: (12-17)=3, (18-25)=4, (>25)=5. GU: (12-20)=3, (>20)=VG.

Problem 1

Consider a rocket moving along the x-axis with constant proper acceleration g, starting at the origin $x = 0$ when $t = 0$. Derive and plot:
i) the relation between x and t
ii) the relation between $v = \frac{dx}{dt}$ and t.
Show that $v < c$ always, and that when $c \to \infty$ the first relation reduces to $x = \frac{1}{2}gt^2$.

Problem 2

A source is sitting at the origin of an inertial frame emitting light at a proper frequency ν_0. An observer travels at constant velocity v on a trajectory parallel to the y-axis and crossing the x-axis at $x = x_0$. Derive the frequency measured when the observer crosses the x-axis and much later when it has moved very far away ($y \to \infty$).

Problem 3

A generic 4-vector can be indicated by its components (v^1, v^2, v^3, v^4), where we will assume for this exercise that $v^{1 \cdots 4}$ are just dimensionless numbers. Give examples of:
i) Two space-like vectors whose sum is time-like.
ii) Two light-like vectors whose sum is time-like.
iii) Two time-like vectors whose sum is time-like.
iv) Two space-like vectors whose sum is space-like.
v) Two Lorentz-orthogonal vectors.

Problem 4

A pion (rest mass $m_\pi = 140$ MeV) hitting a proton at rest (rest mass $m_p = 938$ MeV) can give rise to a kaon (rest mass $m_K = 498$ MeV) and a Lambda particle (rest mass $m_\Lambda = 1116$ MeV). What is the minimum energy the pion must have for this process to be allowed?
(You can either give the total energy E_π or the "kinetic" energy $T_\pi = E_\pi - m_\pi$. Work in $c = 1$ units.)

Problem 5

We want to design a laser-propelled rocket that, after accelerating from rest to its maximum velocity, retains at least half of its original mass. What is the maximum velocity the rocket can reach?

Problem 6

Consider an inertial frame with coordinates (x, y, z, t). From the expression of the Lienard-Wiechert potential

$$\Phi^\mu = q\frac{U^\mu}{R_\nu U^\nu},$$

where U^μ is the 4-velocity of the particle and R^μ the null-vector connecting the observer to the particle, derive the expression for the Coulomb potential $V \equiv \Phi^4$ at the origin due to a particle of charge q moving according to $x = u_0 t,\ y = y_0,\ z = 0$. The velocity u_0 and the position y_0 along the y-axis are constants.

PROBLEM 1

i) $\left(x + \frac{c^2}{g}\right)^2 - c^2t^2 = \frac{c^4}{g^2}$

(Derivation in book and lecture notes)

Note: • units OK.

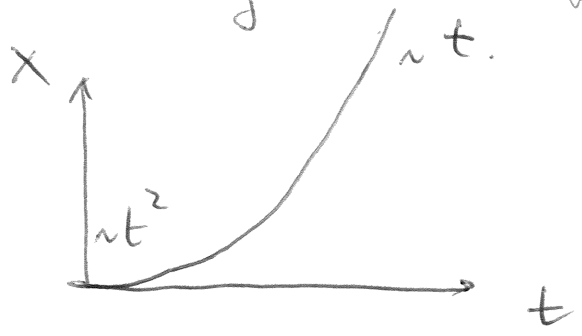
• $t = 0 \Rightarrow x = 0$ OK.

$\Rightarrow x = -\frac{c^2}{g} + \sqrt{\frac{c^4}{g^2} + c^2t^2}$ (+$\sqrt{\ }$ sign !).

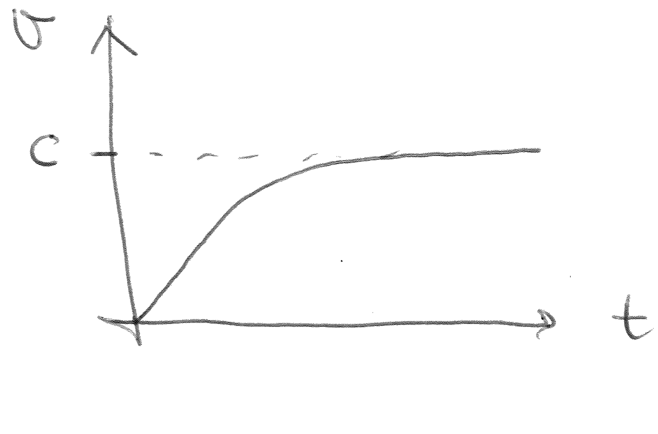
expanding in t: $x = -\cancel{\frac{c^2}{g}} + \cancel{\frac{c^2}{g}} + \frac{1}{2}gt^2 + \dots$

ii)

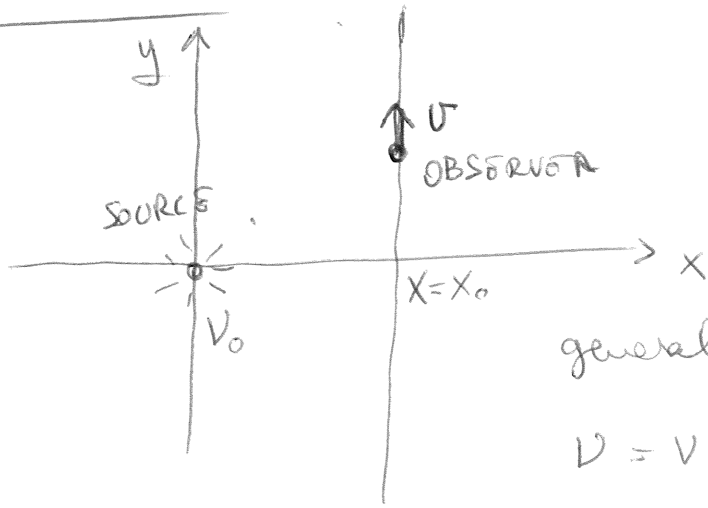
$v = \frac{dx}{dt} = \frac{c^2 t}{\sqrt{\frac{c^4}{g^2} + c^2t^2}} = \frac{gt}{\sqrt{1 + \frac{g^2t^2}{c^2}}} \simeq gt + \dots$



$\lim_{t \to \infty} v = c$



PROBLEM 2



general formula

$$\nu = \nu_0 \frac{1 - v_r/c}{\sqrt{1 - v^2/c^2}}$$

when the observer crosses the x axis the velocity is $\perp$ to the direction of the source. $(v_r = 0)$

$$\Rightarrow \nu = \nu_0 \frac{1}{\sqrt{1 - v^2/c^2}}$$

when the observer is very far away the velocity is almost $\parallel$ to the direction of the source. $(v_r = v)$

$$\Rightarrow \nu = \nu_0 \frac{1 - v/c}{\sqrt{1 - v^2/c^2}} = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

PROBLEM 3

$$V^\mu V_\mu = -(V^1)^2 - (V^2)^2 - (V^3)^2 + (V^4)^2 \begin{cases} > 0 & \text{TIME-like} \\ = 0 & \text{LIGHT-like} \\ < 0 & \text{SPACE-like} \end{cases}$$

i) $(2, 0, 0, 1)$ and $(-2, 0, 0, 1)$

ii) $(1, 0, 0, 1)$ and $(-1, 0, 0, 1)$

iii) $(0, 0, 0, 1)$ and $(0, 0, 0, 2)$

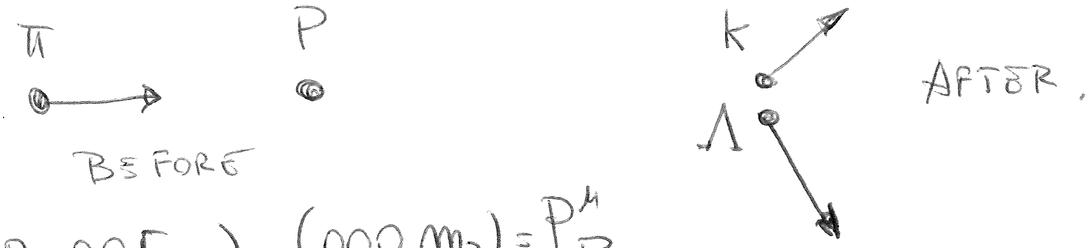
iv) $(1, 0, 0, 0)$ and $(0, 1, 0, 0)$

v) $(1, 0, 0, 2)$ and $(2, 0, 0, 1)$

($V^\mu U_\mu = 0$ means U, V Lorentz orthogonal)

PROBLEM 4

$$\boxed{\pi + P \to k + \Lambda}$$



$$\underbrace{P^\mu_\pi = (P_\pi, 0, 0, E_\pi) \quad (0\,0\,0\,m_p) = P^\mu_p}_{\text{in the LAB frame}}$$

The MINIMUM energy of k and Λ in the CM frame is their rest mass:

$$\underbrace{P^\mu_K = (0\,0\,0\,m_K) \quad P^\mu_\Lambda = (0\,0\,0\,m_\Lambda)}_{\text{IN the CM frame.}}$$

$$\underbrace{(P^\mu_\pi + P^\mu_p)^2 = (P^\mu_K + P^\mu_\Lambda)^2}_{\text{Lorentz invariant, valid in ANY FRAME.}}$$

$$(E_\pi + m_p)^2 - P_\pi^2 = (m_K + m_\Lambda)^2$$

$$m_p^2 + E_\pi^2 + 2 m_p E_\pi - P_\pi^2 = (m_K + m_\Lambda)^2$$

$$m_p^2 + 2 m_p E_\pi + m_\pi^2 = (m_K + m_\Lambda)^2$$

$$E_\pi = \frac{(m_K + m_\Lambda)^2 - m_\pi^2 - m_p^2}{2 m_p} = 909 \text{ MeV.}$$

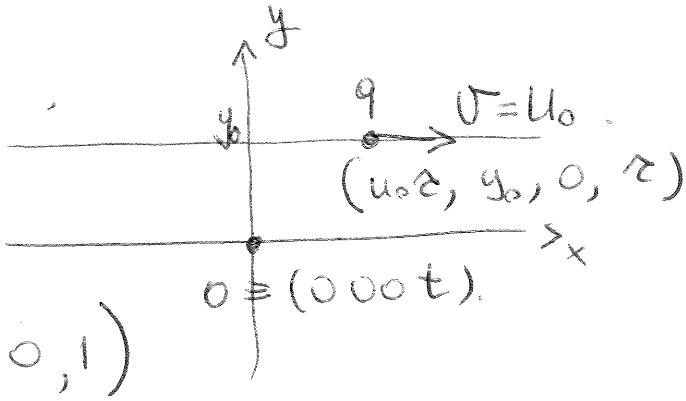
PROBLEM 5.

From the solution to the relativistic Rocket eq. (se notes for derivation).

$$\frac{M_i}{M_f} = \left(\frac{c+v}{c-v}\right)^{\frac{1}{2}}$$

setting $\frac{M_i}{M_f} = 2 \Rightarrow v = \frac{3}{5}c.$

PROBLEM 6



$$U^\mu = \gamma(u_0)\left(u_0, 0, 0, 1\right)$$

$$R^\mu = (-u_0\tau, -y_0, 0, t-\tau)$$

where τ is the time at which the charge affects the origin:

$$R_\mu R^\mu = (t-\tau)^2 - u_0^2 - u_0^2\tau^2 = 0$$

$$\Rightarrow \tau = \frac{t - \sqrt{t^2 - (1-u_0^2)(t^2-y_0^2)}}{1-u_0^2}$$

$$\phi^4 = q\,\frac{\gamma}{\gamma(t-\tau) + \gamma u_0^2\tau} = \frac{q}{t-\tau+u_0^2\tau}$$

where τ is given by the eq. above.

CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045/FYP302 - Speciell Relativitetsteori. 2018-04-04

Examinator: Gabriele Ferretti rum: Soliden S3039
tel. 7723168, 0721582259 email: ferretti@chalmers.se

OBS: Nästa granskningstillfälle: 2018-04-24, 16:00-17:00 i Origo N6115

Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook

Tentamen innehåller 6 uppgifter. Varje uppgift ger max 5 poäng.
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Problem 1

A particle A of rest-mass M_A decays into particles B and C of rest-masses M_B and M_C. Find the total energy, kinetic energy and momentum of the decay products. Could you give such a definite answer if there were more than two decay products? If not, why not? Can you still say something?

Problem 2

The average distance of the Earth from the Sun is about 150 million kilometers. The ecliptic latitude of the star Capella is approximately 22.5 degrees. Discuss the aberration of the light coming from Capella during the year.

Problem 3

Two particles move along the x axis of an inertial frame S at velocities $0.6c$ and $0.7c$, with the fastest one starting 1 m behind. How many seconds in S does it take before the fastest one overtakes the slowest one?

A rod of proper length 1 m moves with velocity $0.6c$ in S. A particle starts from one end of the rod, with velocity $0.7c$ in S in the same direction as the rod. How long does it take for the particle to pass the rod?

Are the two answers the same or different and why?

v.g.v

Problem 4

Consider a head-on elastic collision of a pion with total energy 100 GeV with a proton at rest. What is the γ factor of the pion before the collision? What is the maximum γ factor the pion can have after the collision? What does that mean for the energy of the outgoing proton?

Proton rest-mass: $M = 0.938\ \mathrm{GeV}/c^2$, pion rest-mass: $m = 0.140\ \mathrm{GeV}/c^2$

Problem 5

In a reference frame S there is a constant magnetic field $\mathbf{B}$ pointing in the z direction and no electric field. Discuss the electric and magnetic fields measured by an observer traveling at constant velocity v:

a) along the x direction

b) along the z direction

Problem 6

At what speed does a clock move if it runs at a rate which is one third of the rate of an identical clock at rest?

At what speed does a clock move *away from you* if *you see it ticking* at a rate which is one third of the rate of an identical clock at rest in your frame?

PROBLEM 1.

$$A \to B\ C \qquad p_A^2 = m_A^2 \text{ etc.}$$

$$p_A^\mu = p_B^\mu + p_C^\mu$$

$$(m_A, 000) = (E_B, \mathbb{P}_B) + (E_C, \mathbb{P}_C)$$

$$\Rightarrow \mathbb{P}_B = -\mathbb{P}_C := \mathbb{P}.$$

$$E_B + E_C = m_A \Rightarrow \sqrt{m_B^2 + \mathbb{P}^2} + \sqrt{m_C^2 + \mathbb{P}^2} = m_A$$

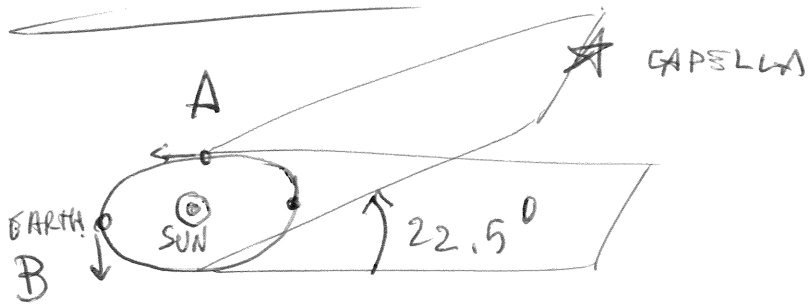
$$\Rightarrow \mathbb{P}^2 = \frac{m_A^2 + m_B^2 + m_C^2 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}{4m_A^2}$$

$$E_{B/C} = \sqrt{m_{B/C} + \mathbb{P}^2}$$

$$T_{B/C} = E_{B/C} - m_{B/C}$$

For $A \to BCD$ I can only say the decay occours on a plane.

PROBLEM 2



Velocity of Earth around Sun:

$$v \simeq \frac{2\pi \times 150 \times 10^{9}\, m}{1\, yr} \simeq 3\times 10^{4}\, m/s$$

$$\frac{v}{c} \simeq 10^{-4} \ll 1.$$

Aberration $\cos\alpha' = \dfrac{\cos\alpha + v/c}{1 + v\cos\alpha/c}$

Taylor: (setting $\alpha' = \alpha + \Delta\alpha$ $\Delta\alpha \ll 1$).

$$\Delta\alpha \simeq -\frac{v}{c}\sin\alpha$$

When the Earth is in A $\alpha = 22{,}5°$

B $\alpha = 90°$.

$$\Rightarrow |\Delta\alpha| = \begin{cases} 10^{-4}\times \sin 22{,}5° \simeq 7{,}89'' \\ 10^{-4} \simeq 20.6'' \quad \text{in B} \end{cases}$$

to convert, multiply by $\frac{180}{\pi}\times 60 \times 60$

PROBLEM 3

In the first case I simply use the "mutual" velocity in S (NO NEED to "add" them). $v_{mutual} = 0.1c$

$$\Rightarrow T = \frac{1m}{v_{mutual}} = 3.3 \times 10^{-8} s.$$

In the second case I must take contraction into account:

$$L_{in\ S} = 1m/\gamma = \sqrt{1-0.6^2} = 0.8m$$

So the time it takes is

$$T = \frac{0.8m}{v_{mutual}} = 2.64 \times 10^{-8} s.$$

PROBLEM 4

$$\gamma_\pi = \frac{E_\pi}{m_\pi} = 714 \qquad \text{before collision}$$

After (see problem 6.7):

$$\gamma_\pi^{max} = \frac{m_\pi^2 + m_p^2}{2 m_\pi m_p} \simeq 3.4$$

Almost all energy is transferred to the proton.

PROBLEM 5 I USE $x^{\mu=0123}=(t\,x\,y\,z)$ notation!

Let B be the z-component.

$$F^{\mu\nu}=\begin{pmatrix}0&0&0&0\\0&0&B&0\\0&-B&0&0\\0&0&0&0\end{pmatrix}$$

Case a): $$\Lambda^{\mu}{}_{\nu}=\begin{pmatrix}\gamma&-\gamma v&0&0\\-\gamma v&\gamma&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

$$F'^{\mu\nu}=\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}F^{\rho\sigma}\left(=\underbrace{(\Lambda F\Lambda^T)^{\mu\nu}}_{\text{as matrices if you want}}\right).$$

$$=\begin{pmatrix}0&0&-v\gamma B&0\\0&0&\gamma B&0\\v\gamma B&-\gamma B&0&0\\0&0&0&0\end{pmatrix}$$

From which: $E'_y=v\gamma B$, $B'_z=\gamma B$.

Similarly case b): $$\Lambda^{\mu}{}_{\nu}=\begin{pmatrix}\gamma&0&0&-v\gamma\\0&1&0&0\\0&0&1&0\\-v\gamma&0&0&\gamma\end{pmatrix}$$

in this case nothing changes!

$\mathbb{E}'=0$ $\mathbb{B}'=\mathbb{B}$. as before.

PROBLEM 6

The first case is about time dilation;

$$\frac{1}{3} = \sqrt{1 - v^2/c^2} \Rightarrow \frac{v}{c} = \frac{2\sqrt{2}}{3} \simeq 0,94$$

The second case is about Doppler eff.

$$\frac{1}{3} = \frac{1 - v/c}{1 + v/c} \Rightarrow \frac{v}{c} = \frac{1}{2}$$

CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045/FYP302 - Speciell Relativitetsteori. 2018-08-24

Examinator: Gabriele Ferretti rum: Soliden S3039
tel. 7723168, 0721582259 email: ferretti@chalmers.se

OBS: Nästa granskningstillfälle: 2018-10-08, 16:00-17:00 i Origo N6115

Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook

Tentamen innehåller 6 uppgifter. Varje uppgift ger max 5 poäng.
Betygsgränser: CTH: (12-17)=3, (18-25)=4, (>25)=5. GU: (12-20)=G, (>20)=VG.

Problem 1

Consider two inertial frames S and S' in standard configuration. (S' moving with velocity v along the x-axis of S.) We learned that time in each frame can be defined by properly synchronizing the clocks of that frame. At a certain time t'_0 in S' all clocks of S' emit a flash of light. Describe this phenomenon in the S frame. Draw a Minkowski diagram for illustration.

Problem 2

The rapidity ϕ of a particle is defined by the formula $e^{\phi} = \gamma(v)(1 + v/c)$. Show that $\tanh\phi = v/c$.

Problem 3

How many successive incremental boosts of 10%c are needed to reach a velocity of 99%c starting at rest?
Hint It's easier to use one of the formulas of Problem 2.

Problem 4

A proton with kinetic energy of 1 GeV collides with a deuteron at rest. One of the possible processes is the production of a number n of pions: $pD \to pD\pi \ldots \pi$.
What is the maximum number n of pions that can be produced?
Proton mass: $m_p = 0.938$ GeV/c^2, Deuteron mass: $m_D = 1.875$ GeV/c^2, Pion mass: $m_\pi = 0.140$ GeV/c^2.
Recall that the kinetic energy is defined as the total relativistic energy minus the rest energy.

v.g.v

Problem 5

Consider the event taking place at $X^\mu \equiv (x, y, z, ct) = (1, 1, 0, 1)$ in some reference frame and for some unit of length. Give an example of another event X'^μ, in the same frame and units, that is

- Space-like separated from X^μ
- Time-like separated from X^μ
- Light-like separated from X^μ
- Lorentz orthogonal to X^μ

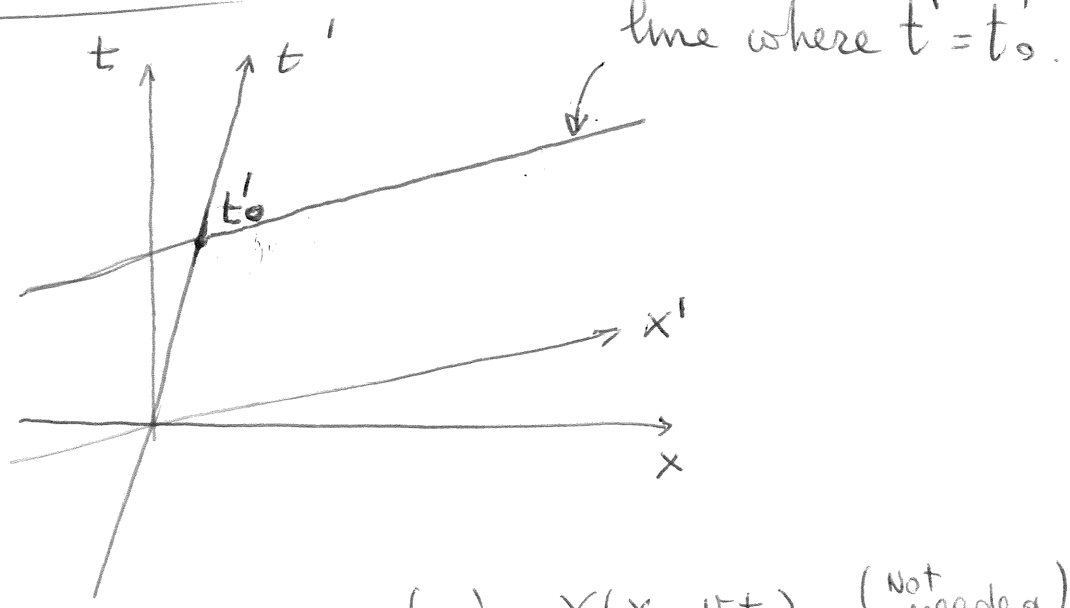
Problem 6

A particle moves with constant velocity $\mathbf{u}$ in an inertial frame S making an angle α with the positive x-axis. An observer moves along the x-axis with velocity v defining a new inertial frame S' in standard configuration.

Derive the "particle aberration formula" giving the new angle α' between the particle direction and the x'-axis in the new frame. Discuss the similarities and differences with other similar formulas that you may know.

Suppose $\alpha = \pi/3$ and $u = c/3$. What is the velocity v required so that the observer in S' sees the particle coming orthogonal to the x'-axis?

Problem 1



Lorentz transf: $\begin{cases} x' = \gamma(x - vt) & \text{(Not needed)} \\ t' = \gamma(t - \frac{v}{c^2}x) \end{cases}$

Setting $t' = t'_0 \Rightarrow t'_0 = \gamma(t - \frac{v}{c^2}x)$

$$\Rightarrow x = \frac{c^2}{v} t - \frac{c^2}{v}\frac{t'_0}{\gamma} = \frac{c^2}{v} t + \text{const.}$$

Same as a "superluminal" signal of "velocity" $\frac{c^2}{v}$ ($> c$ always).

It's ok because it is not a "real" signal.

PROBLEM 2

$$\sinh\phi = \frac{e^{\phi} - e^{-\phi}}{2}, \quad \cosh\phi = \frac{e^{\phi} + e^{-\phi}}{2}$$

$$\Rightarrow \tanh\phi = \frac{e^{\phi} - e^{-\phi}}{e^{\phi} + e^{-\phi}} = \frac{\gamma(1+\frac{v}{c}) - \frac{1}{\gamma(1+v/c)}}{\gamma(1+\frac{v}{c}) + \frac{1}{\gamma(1+\frac{v}{c})}} =$$

$$= \frac{\gamma^2(1+\frac{v}{c})^2 - 1}{\gamma^2(1+\frac{v}{c})^2 + 1} = \frac{\frac{(1+v/c)^2}{1-v^2/c^2} - 1}{\frac{(1+v/c)^2}{1-v^2/c^2} + 1} =$$

$$= \frac{\frac{1+\frac{v}{c}}{1-v/c} - 1}{\frac{1+v/c}{1-v/c} + 1} = \frac{1+\frac{v}{c} - (1-\frac{v}{c})}{1+\frac{v}{c} + 1-\frac{v}{c}} =$$

$$= \frac{2v/c}{2} = \frac{v}{c}.$$

PROBLEM 3

$\phi = \operatorname{arctanh}\left(\frac{v}{c}\right)$ is ADDITIVE

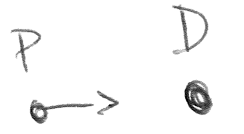
$\Rightarrow \phi(v_n) = n\,\phi(v_1)$

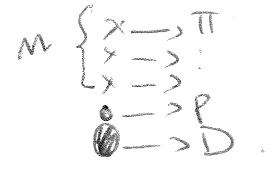
$\Rightarrow \phi(0,99) = n\,\phi(0,10)$

$\Rightarrow \operatorname{arctanh}(0,99) = n \operatorname{arctanh}(0,10)$

$$\Rightarrow \quad n = \frac{\operatorname{arctanh}(0,99)}{\operatorname{arctanh}(0,10)} = \frac{2,647}{0,100} \simeq 26.$$

PROBLEM 4. $\boxed{\text{Set } c=1}$





$$s = (P_p^\mu + P_D^\mu)^2 > (m_p + m_D + n\, m_\pi)^2$$

$$P_p^\mu = (p, 0, 0, E_n) \qquad P_D^\mu = (0\,0\,0\, m_D).$$

$$s = -p^2 + (E_n + m_D)^2 = -p^2 + E_n^2 + 2 m_D E_n + m_D^2$$

$$= 2 m_D E_n + m_D^2 + m_p^2$$

$$\Rightarrow E_n > \frac{(m_p + m_D + n\, m_\pi)^2 - m_p^2 - m_D^2}{2 m_D}.$$

numerically $n \leq 4$.

$(E_n = 1\,\text{GeV} + m_p c^2)$.

PROBLEM 5

$$(x^\mu - x'^\mu)^2 \begin{cases} > 0 & \text{time-like} \\ = 0 & \text{Light-like} \\ < 0 & \text{Space-like} \end{cases}$$

$x^\mu x'_\mu = 0$ Lorentz orthogonal.

Examples (there are ∞ many!).

Space like : $x'^\mu = (0\,0\,0\,1)$

Time like : $x'^\mu = (1\,1\,0\,0)$

Light like : $x'^\mu = (1\,0\,0\,0)$

Lorentz $\perp$: $x'^\mu = (1\,1\,0\,2)$

PROBLEM 6.

Particle aberration formula

$$\tan\alpha' = \frac{\sin\alpha}{\gamma(v)\left(\cos\alpha - \frac{v}{u}\right)}$$

(For the proof, see ex. 4.13 in Rindler)

The direction is $\perp$ when $\alpha' = \frac{\pi}{2}$

$\Rightarrow \tan\frac{\pi}{2} = \infty \quad \Rightarrow \quad \cos\alpha = \frac{v}{u}$

$\Rightarrow v = u\cos\alpha = \frac{c}{3} \cdot \cos\frac{\pi}{3} = \frac{1}{6} \cdot c$

CHALMERS TEKNISKA H?GSKOLA och G?TEBORGS UNIVERSITET

FUF045 - Speciell Relativitetsteori
Examinator: Gabriele Ferretti   rum: Origo 6109
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OBS: N?sta granskningstillf?lle: 2016-02-19, 15:00-18:00 i mitt rum.
(Denna information st?r ?ven p? hemsidan. Tentorna kommer att finnas hos mig.)

Hj?lpmedel:

- Chalmersgodk?nd minir?knare.
- Physics Handbook
- Kursboken "Rindler"
  (F?r studenter som har k?pt pdf filen g?r det bra att skriva ut de relevanta sidorna.)

Tentamen inneh?ller 3 uppgifter. Varje uppgift ger max 10 po?ng.

Problem 1

A particle moves with constant velocity $\mathbf{u}$ in an inertial frame S making an angle α with the positive x-axis. An observer moves along the x-axis with velocity v defining a new inertial frame S' in standard configuration.

Derive the "particle aberration formula" giving the new angle α' between the particle direction and the x'-axis in the new frame. Discuss the similarities and differences with other similar formulas that you may know.

Suppose $\alpha = \pi/6$ and $u = c/2$. What is the velocity v required so that the observer in S' sees the particle coming orthogonal to the x'-axis?

Problem 2

What is the minimal (threshold) energy that a photon γ must have so that when it collides with a proton p at rest it gives rise to a final state consisting of a proton and a neutral pion π^0: $\gamma + p \to \pi^0 + p$? (This is called the Primakoff effect).

Express the photon energy in MeV, knowing that the mass of the proton is 938 MeV/c^2 and the mass of the pion is 135 MeV/c^2.

For photon energy just above threshold how do the final proton and pion move relative to each other?

Problem 3

A spaceship propels itself by emitting radiation in the direction opposite to its motion. It starts at rest and ends up with a velocity $v = 1.2 \times 10^8$ m/s in the initial rest-frame.

Determine the ratio between the initial and final rest-masses. Discuss the subject from a broad perspective.

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OBS: Nästa granskningstillfälle: 2016-04-29, 15:00-16:30 i mitt rum.
(Tentorna finns hos mig.)

Hjälpmedel:

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Problem 1

5 p. A spaceship travels at a constant speed to a star a distance n light-years away in a time its crew considers to be m years. What is the speed of the spaceship? (Note, to get all points you must derive the formula.)
3 p. Are there any limits on the values m and n can take?
2 p. What is the speed if the distance is $4. \times 10^{13}$ km and for the crew it takes 10 years to get there?

Problem 2

2 p. Derive and discuss briefly (max 1 page!) the concept of proper acceleration.
4 p. An electron moves in an inertial frame according to the equations $x = \omega t$, $y = A \sin \omega t$ and $z = 0$ for $t > 0$. Find the proper acceleration as a function of t.
2 p. Are there any constraints on the values ω and A can take?
2 p. Another observer moves with constant velocity u along the z axis. What is the proper acceleration of the electron in the new observer's frame?

Problem 3

3 p. Discuss briefly (max 1 page!) how conservation of momentum and energy differ in Newtonian mechanics and special relativity.
5 p. The subatomic particle Λ, of mass 1116. MeV decays at rest into a proton (mass 938. MeV) and a pion (mass 140. MeV) Find the energies of the outgoing proton and pion.
2 p. Find also the velocities (in units of c) of the outgoing proton and pion.

CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045 - Speciell Relativitetsteori

Examinator: Gabriele Ferretti   rum: Soliden S3039
tel. 7723168, 0721582259   email: ferretti@chalmers.se

OBS: Nästa granskningstillfälle: 2016-09-05 14:00-15:00 i Origo N-6115.
(Jag tar tentorna med mig.)

Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook
- Kursboken "Rindler"
  (För studenter som har köpt pdf filen går det bra att skriva ut de relevanta sidorna.)

Tentamen innehåller 3 uppgifter. Varje uppgift ger max 10 poäng.

Problem 1

A distant star ejects a jet of gas towards the Earth at speed v and at an angle θ with the line of sight. An astronomer (incorrectly) interprets this jet as moving at a certain apparent speed V perpendicularly to the line of sight. Find the expression of V as a function of v and θ. For what values of v and θ can the apparent speed be larger than c? Discuss.

Problem 2

A "light clock" consists of two mirrors at the end of a short rod with a photon bouncing between them. Assuming length contraction and the constancy of the speed of light, prove that such clock will go slow by the expected Lorentz factor if it travels (a) longitudinally, (b) transversely through an inertial frame. What can you say about the case where the motion is neither transverse nor longitudinal?

Problem 3

A Higgs boson at rest decays into two photons.
One of the photons has 3-momentum $\mathbf{p} = (31.25, 54.13, 0)$ GeV/c. What is the 4-momentum of the photon? What is the 4-momentum of the other photon? Compute the mass of the Higgs boson in units of GeV/c^2 using this data. Discuss!

CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045/FYP302 - Speciell Relativitetsteori. 2017-01-10

Examinator: Gabriele Ferretti rum: Soliden S3039
tel. 7723168, 0721582259 email: `ferretti@chalmers.se`

OBS: Nästa granskningstillfälle: 2017-02-03 15:00-16:00 i Origo rum 6115.
Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook

Tentamen innehåller 6 uppgifter. Varje uppgift ger max 5 poäng.

Problem 1

Prove (in any way you want) the relativistic formula for addition of velocities:

Let S_1, S_2 and S_3 three inertial systems in "standard configuration". Let v_{12} be the velocity of S_2 with respect to S_1 and v_{23} the velocity of S_3 with respect to S_2. Find v_{13}, that is the velocity of S_3 with respect to S_1. (Note that you have to prove this, not just write down the formula.)

Problem 2

In a given inertial frame S, two particles are shot out simultaneously from a given point with equal speeds v in two directions making an angle *of 30 degrees* with each others in S. What is the speed of each particle relative to the other?

Problem 3

Electrons at the LEP accelerator (a storage ring of radius 4.3 km) were circulating with a gamma-factor of $\gamma = 2. \times 10^5$. What was their proper acceleration?

Problem 4

How fast must a proton move before its kinetic energy equals ten times its rest energy?

v.g.v

Problem 5

Two photons with the same frequency ν travel in an inertial frame S, one along the positive x-axis and the other along the positive y-axis. Find the velocity of the center of mass frame (also called "zero momentum frame").

Problem 6

A charged pion π^+ (of mass 140 MeV/c^2) decays into a muon μ^+ (of mass 105 MeV/c^2) and an approximately massless neutrino ν_μ: $\pi^+ \rightarrow \mu^+ \, \nu_\mu$. Find the energies of the muon and of the neutrino in the pion's rest frame.

CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045/FYP302 - Speciell Relativitetsteori. 2017-04-10

Examinator: Gabriele Ferretti rum: Soliden S3039
tel. 7723168, 0721582259 email: `ferretti@chalmers.se`

OBS: Nästa granskningstillfälle: mejla mig f́or att boka en tid om ni vill.
Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook

Tentamen innehåller 6 uppgifter. Varje uppgift ger max 5 poäng.

Problem 1

A spaceship travels at constant speed to a star 10 light years away in a time that its crew considers to be 5 years. The ship then turns around and comes back to Earth at the same speed. How long time does the whole round trip take from Earths point of view?

Problem 2

A particle moves at uniform speed u around a circular path of radius r. *Derive in detail* the expression for the proper acceleration α.

Problem 3

A source of monochromatic light of proper frequency ν_0 is fixed at the origin of an inertial frame S. An observer travels through S with velocity u of radial component u_r relative to the source. What frequency does the observer see? In what sense is this different from the usual Doppler effect where it is the light source that moves?

Problem 4

A rocket propels itself by giving out some of its mass in form of exhaust with constant backward velocity U relative to the rocket instantaneous rest frame. The rocket starts at rest in a given inertial frame and reaches a velocity V. Discuss the differences between the relativistic and non-relativistic case. What is conserved and what is not? What is the optimal velocity U that minimizes the quantity of additional mass that needs to be carried? Find an equation for the ratio M_i/M_f between the initial and final mass and discuss its non-relativistic limit.

(v.g.v.)

Problem 5

Consider a head-on elastic collision between a "bullet" of mass M hitting a stationary "target" of mass m. Prove that, after the collision, the energy of the bullet cannot exceed a certain value and find that value. What are the similarities/differences with the non-relativistic case?

Problem 6

Consider the reaction $\pi^- + p \to K^0 + \Sigma^0$, where a pion π^- hits a proton p at rest and produces a kaon K^0 and a Σ^0 baryon. What is the minimum total and kinetic energy that the pion must have in the proton rest frame for this reaction to be possible?

The masses are, in units of MeV$/c^2$, $m(\pi^-) = 140$, $m(p) = 938$, $m(K^0) = 498$, $m(\Sigma^0) = 1193$.

CHALMERS TEKNISKA HÖGSKOLA och GÖTEBORGS UNIVERSITET

FUF045/FYP302 - Speciell Relativitetsteori. 2017-08-18

Examinator: Gabriele Ferretti   rum: Soliden S3039
tel. 7723168, 0721582259   email: `ferretti@chalmers.se`

OBS: Nästa granskningstillfälle: 2017-09-15, 16:00-17:00 i Origo N6115

Hjälpmedel:

- Chalmersgodkänd miniräknare.
- Physics Handbook

Tentamen innehåller 6 uppgifter. Varje uppgift ger max 5 poäng.

Problem 1

Two particles move along the x axis of an inertial frame S at velocities $0.6c$ and $0.7c$, with the fastest one starting 1 m behind. How many seconds in S does it take before the fastest one overtakes the slowest one?

A rod of proper length 1 m moves with velocity $0.6c$ in S. A particle starts from one end of the rod, with velocity $0.7c$ in S in the same direction as the rod. How long does it take for the particle to pass the rod?

Are the two answers the same or different and why?

Problem 2

The average distance of the Earth from the Sun is about 150 million kilometers. The ecliptic latitude of the star Vega is approximately 61 degrees. Discuss the aberration of the light coming from Vega during the year.

Problem 3

A particle moves in an inertial frame (x, t) according to the equation $x = (k/3)t^3$ for some constant k. Derive the proper acceleration of the particle. Can the equation above be valid for all times?

Problem 4

A particle A of rest-mass M_A decays into particles B and C of rest-masses M_B and M_C. Find the total energy, kinetic energy and momentum of the decay products. Could you give such a definite answer if there were more than two decay products? If not, why not? Can you still say something?

v.g.v

Problem 5

Consider a head-on elastic collision of a pion with total energy 100 GeV with a proton at rest. What is the γ factor of the pion before the collision? What is the maximum γ factor the pion can have after the collision? What does that mean for the energy of the outgoing proton?

Proton rest-mass: $M = 0.938$ GeV$/c^2$, pion rest-mass: $m = 0.140$ GeV$/c^2$

Problem 6

What are the Lienard-Wiechert potentials? Give a derivation with a clear definition of the quantities involved.