Storgruppsövning II

<u>Tenta 18-04-05</u> 3) Beräkna $\lim_{x\to 0} \dfrac{1-(\cos x)^{\sin x}}{x^3}$

$$1-(\cos x)^{\sin x} = 1 - e^{\sin x \cdot \ln \cos x} = 1 - e^{(x - x^3/6 + O(x^5))\ln(1 - x^2/2 + O(x^4))}$$

$$= 1 - e^{\underbrace{(x - x^3/6 + O(x^5))((-x^2/2 + O(x^4)) + O((-x^2/2 + O(x^4))^2))}_{= -x^3/2 + O(x^4)}}$$

$$= 1 - e^{-x^3/2 + O(x^4)} = 1 - (1 + (-x^3/2 + O(x^4)) + O(x^6))$$

$$= + x^3/2 + O(x^4).$$

$$\rightsquigarrow \quad \frac{1-(\cos x)^{\sin x}}{x^3} = \frac{+x^3/2 + O(x^4)}{x^3} = +\frac{1}{2} + O(x) \to +\underline{\underline{\frac{1}{2}}}, \text{ då } x \to 0^+.$$

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<u>Tenta 18-08-23</u> 3) $\lim_{x\to 0}\left(\dfrac{1}{\ln(x+\sqrt{1+x^2})} - \dfrac{1}{\ln(1+x)}\right) = ?$

$$= \frac{\ln(1+x) - \ln(x+\sqrt{1+x^2})}{\ln(x+\sqrt{1+x^2})\cdot \ln(1+x)} \qquad (*)$$

- $\sqrt{1+y} = 1 + \frac{1}{2}y - \frac{1}{4}y^2 + O(y^3)$ ← [Kontrollera detta!]

$$\rightsquigarrow \sqrt{1+x^2} = 1 + \tfrac{1}{2}x^2 + O(x^4) \rightsquigarrow \ln(x+\sqrt{1+x^2}) = \ln(1 + x + \tfrac{1}{2}x^2 + O(x^4))$$

$$= x + \tfrac{1}{2}x^2 - \tfrac{1}{2}(x + \tfrac{1}{2}x^2)^2 + O(x^3)$$

$$= x + O(x^3)$$

- $\ln(1+x) = x - x^2/2 + O(x^3)$

$$\rightsquigarrow (*) = \frac{(\cancel{x} - x^2/2 + O(x^3)) - (\cancel{x} + O(x^3))}{(x + O(x^3))\cdot(x - x^2/2 + O(x^3))} = \frac{-\frac{1}{2}x^2 + O(x^3)}{x^2 + O(x^3)}$$

$$= \frac{-1/2 + O(x)}{1 + O(x)} \to -\underline{\underline{\frac{1}{2}}}, \text{ då } x \to 0^+.$$

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Tenta 17-01-13: 3) $f(x) = \sin(x^2)\cos(x)\ln(1+x^2)$

Bestäm $f^{(6)}(0)$.

Entydighet (Föreläsning IV)

Idé: Skriv $f(x) = \sum_{k=0}^{6} a_k x^6 + O(x^7) \rightsquigarrow a_6 = \frac{f^{(6)}(0)}{6!}$

- $\sin x = x - x^3/6 + x^5/120 + O(x^7) \rightsquigarrow \sin x^2 = x^2 - x^6/6 + O(x^{10})$
- $\cos x = 1 - x^2/2 + x^4/24 + O(x^6)$
- $\ln(1+x) = x - x^2/2 + O(x^3) \rightsquigarrow \ln(1+x^2) = x^2 - x^4/2 + O(x^6)$

$$\rightsquigarrow \sin(x^2)\cdot\cos(x)\cdot\ln(1+x)$$

$$= \underbrace{\underbrace{\left(x^2 - x^6/6 + O(x^{10})\right)\left(1 - x^2/2 + x^4/24 + O(x^6)\right)}_{= x^2 - x^4/2 + O(x^6)}\left(x^2 - x^4/2 + O(x^6)\right)}$$

$$= x^4 - x^6/2 + O(x^8) - x^6/2 + O(x^8)$$

$$= x^4 - x^6 + O(x^8) \rightsquigarrow a_6 = -1$$

$$\rightsquigarrow \boxed{f^{(6)}(0) = -6!}$$

Tenta 13-12-18 3a) Låt $f(x) = \frac{1}{1+\frac{1}{2}x^2}$. Bestäm Taylorpolynomet av grad ≤ 7.

Geometrisk summa:

$$\frac{1}{1-t} = 1 + t + t^2 + \ldots + t^n + \overbrace{\frac{t^{n+1}}{1-t}}^{= O(t^{n+1})} \quad \forall t \neq 1.$$

$$t = -\frac{1}{2}x^2 \rightsquigarrow \frac{1}{1+\frac{1}{2}x^2} = \underbrace{1 - \frac{1}{2}x^2 + \frac{1}{4}x^4 - \frac{1}{8}x^6}_{= p(x)} + O(x^8)$$

Taylorpolynomet av grad ≤ 7

(enl. entydighetssatsen — Föreläsning IV)

Tenta 14-08-21 : 3) $\lim_{x\to\infty} e^x\left(\ln(1+e^x)-x\right)$

$= \{y=e^x\to\infty,\ x\to\infty\} = \lim_{y\to\infty} y\left(\underbrace{\ln(1+y)-\ln y}_{=\ln\left(1+\frac{1}{y}\right)}\right)$

$= \{t=1/y\to 0,\ y\to\infty\} = \lim_{t\to 0^+} \frac{\ln(1+t)}{t} = \lim_{t\to 0^+} \frac{\frac{1}{1+t}}{1} = \underline{\underline{1}}.$

l'Hospital!