A project on the American put option

The American put option with strike K and maturity T gives the owner the right to sell the underlying stock for the price K at any time $t \in (0, T]$. The binomial model is the most popular numerical method to compute the fair price of American puts.

The main goal of this project is to perform a parameter sensitivity analysis of the American put and to derive numerically the optimal exercise curve of the option using the binomial model and Matlab. It is assumed that the underlying stock pays no dividend. The specific goals of the project are the following.

- Study numerically how the value of the American put at time $t = 0$ depends on the parameters σ, r, K, T and give an intuitive explanation of the results obtained.
- Derive numerically the curve $(t_i, S_*(t_i))$ where t_i is an optimal exercise time and $S_*(t_i)$ is the largest stock price for which it is optimal to exercise the American put at time t_i (optimal exercise curve). Describe the qualitative properties of this curve.
- Show numerically that the binomial price at time $t = 0$ of the American put with strike K and maturity T converges, as $T \to +\infty$, to the value $v(S_0)$, where v is the function

$$v(x) = \begin{cases} K - x & 0 \le x \le L \\ (K - L)\left(\frac{x}{L}\right)^{-\frac{2r}{\sigma^2}} & x > L \end{cases}, \quad L = \frac{2r}{2r + \sigma^2} K.$$

  (The American put with infinite maturity is called perpetual American put. Of course, it does not exist in real markets but it is useful for theoretical considerations.)

The results of the project have to be outlined in a report consisting of three sections:

- The first section is the Introduction (1-2 pages). Here is where you state the problem, describe the theoretical framework and discuss the financial applications.
- The second section is where you report on the goals of the project (3-4 pages).
- The third section is the Conclusion. Summarize your most interesting results in this section (1 page).

Remarks:

- The report has to be submitted in PDF form by e-mail to calogero@chalmers.se. Use OPTIONS 2020 as subject and include all members of the group as recipients. Attach your Matlab codes to the e-mail (preferably in a ZIP file).
- The deadline for submission is **January 3rd, 2020**. The grade on the project (max 2 bonus points) will be communicated on Tuesday January 7th.
- For help on the project you can pass to my office (L2036) during my office hours (Tuesday 13.30-15, Friday 13.30-15). Further assistance will be provided in the lecture on December 18th (bring your laptop!). This lecture will take place in the room MVF33 and *not* in the usual room (Pascal).