Exam for the course "Options and Mathematics" (CTH[*MVE095*], GU[*MMG810*]) 2019/20

Telefonvakt/Rond: ??

This is a sample exam!

REMARKS: (1) No aids permitted (2) Minor errors in the calculations will be forgiven, but remember that fractions look nicer when you simplify them!

Part I

1. Assume that the market is frictionless, arbitrage free and that the assets pay no dividend. Prove that the value of call options is a non-increasing and convex function of the strike price (max. 2 points).

2. Derive the Black-Scholes price of European call options (max. 2 points).

3. Give and explain the definition of optimal exercise time of American put options (max. 2 points).

4. Decide whether the following statements are true or false an explain your answer (max. 2 points):

   (a) In a frictionless, arbitrage-free market with positive risk-free rate the value of European put options is non-decreasing with maturity.

   (b) In a frictionless, arbitrage-free market with positive risk-free rate the value of American put options is non-decreasing with maturity.

Part II

1. A European derivative on a stock pays the amount

$$Y = (\min(S(T) - 10, 20 - S(T), 2))_+ - 1.$$

   Draw the pay-off function of the derivative and find a constant portfolio of European call/put options that replicates the derivative (max 4 points).

2. Consider a binomial market with parameters $e^u = \frac{7}{4}$, $e^d = \frac{1}{2}$, $S(0) = 1$, $p = 3/4$, $e^r = 9/8$.

a) Compute the binomial price at $t = 0, 1, 2$ of an American put with strike $K = 3/4$ and maturity $T = 2$ (max. 1 point).

b) Compute the binomial price at $t = 0, 1, 2$ of a European call with strike $K = 3/4$ and maturity $T = 2$ (max. 1 point).

c) A derivative $\mathcal{U}$ gives to its owner the right to convert $\mathcal{U}$ at time $t = 1$ into either the European call or the American put defined above. Compute the binomial price of $\mathcal{U}$ at time $t = 0$ (max. 1 point).

d) Describe the strategy that maximizes the expected return for the holder of $\mathcal{U}$ (max. 1 point).

3. The European physically settled digital put option is the derivative with pay-off $Y = S(T)H(K - S(T))$ at maturity T, where $S(t)$ is the price of the underlying stock, K is the strike price of the option and $H(z)$ is the Heaviside function. Derive the Black-Scholes price of this derivative and the number of stock shares in the hedging portfolio (max 4 points).