Theory Questions

The first part of the written exam consists of two theory questions. One of these theory questions will be selected from the following list. To help you get started solving the theory questions, we have provided solutions to T1 and T2. Note that the first part of the written exam is a closed book exam, and no aids are thus permitted in this part.

T1 – Definitions of relative velocities and angles

Derive a relation between the velocity components, the absolute inflow angle α and the relative inflow angle β.

Solution:

The relative velocity vector is defined as the vector subtraction of the blade velocity vector from the absolute velocity vector

$$\boldsymbol{w} = \boldsymbol{c} - \boldsymbol{U}$$

This relation may also be written in terms of its vector components as follows

$$w_x = c_x$$

$$w_r = c_r$$

$$w_\theta = c_\theta - U \qquad (1)$$

Note that the blade only moves in the circumferential direction and $\boldsymbol{U}$ therefore only has a tangential velocity component. It is also important to note that we make no assumptions on the sign of the different vector components in the above expression. In other words, we may have that $c_\theta < 0$ for example. Let us now define the meridional velocity as the velocity in the meridional plane (also known as the axial-radial plane). From Figure 1 (a) it can be seen that the meridional velocity (denoted c_m.) may be calculated as follows

$$c_m = \sqrt{c_x^2 + c_r^2}$$

The absolute flow angle is defined as the angle between the absolute velocity vector and the meridional velocity (see Figure 1 (c))

$$\tan\alpha = \frac{c_\theta}{c_m} \qquad (2)$$

The relative flow angle is further defined as the angle between the relative velocity vector and the meridional velocity (see Figure 1 (c)):

$$\tan\beta = \frac{w_\theta}{c_m} \qquad (3)$$

By combining (1), (2) and (3) one finally obtains the sought relation

$$\tan\beta = \tan\alpha - \frac{U}{c_m}$$

The above expressions are defined for a cylindrical coordinate system. It is important to note that the book often uses a different sign convention that ensures that properties such as circumferential velocity and flow angles remain positive, see p. 4 in Dixon and Hall.

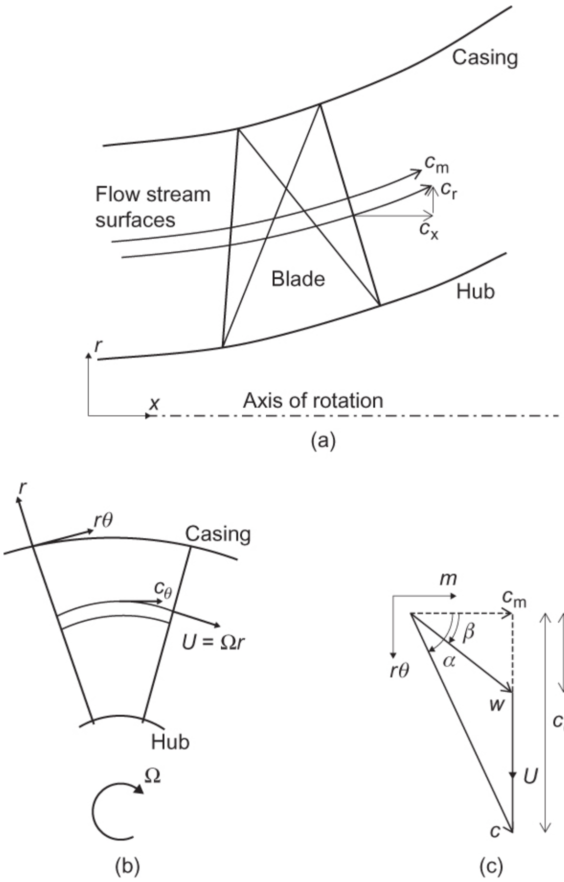

Figure 1: Coordinate system for a general turbomachine.

T2 – Rothalpy

Show that the rothalpy (I) is constant throughout a turbomachine.

Solution:

According to the course literature it is permissible to assume that gravitational effects can be neglected for most turbomachines. The literature also states that the flow through most turbomachines can be considered adiabatic. Under these assumptions, the first law of thermodynamics may be written as follows

$$-\dot{W}_x = \dot{m}\,(\,h_{02} - h_{01}\,) \tag{4}$$

Here, subscripts 1 and 2 respectively denote the inlet and outlet of the control volume. Let us take this control volume to be an axi-symmetric streamtube that passes over a turbomachinery blade. A streamtube is aligned with the meridional velocity of the flow, and the fluid can thus only enter to the left and leave to the right. The left boundary thus becomes the inlet and the right boundary the outlet (and we denote these boundaries with subscripts 1 and 2 respectively). An illustration of a stream surface (upper or lower boundary of our streamtube) is shown in Figure 2. The rate at which a turbomachinery blade adds work to the fluid inside the control volume is further

$$-\dot{W}_x = \tau_a \Omega \; = \dot{m}(r_2 c_{\theta 2} - r_1 c_{\theta 1})\Omega = \; \dot{m}(U_2 c_{\theta 2} - U_1 c_{\theta 1}) \tag{5}$$

The first equality in the above expression follows from the fact that the torque is defined as the force on the blade times the radius of the blade. Thus, the work done by the blade ($Force * Velocity$) becomes $\tau_a \Omega$ since the velocity of the blade equals the radius of the blade times the rotational speed of the blade. The second equality in the above expression follows form the conservation of angular momentum ($Torque = Rate\ of\ Change\ in\ Angular\ Momentum$). Finally, the third equality in the above expression is obtained by multiplying the rotational speed with the radius to obtain the speed of the blade. We may now equate (4) and (5) to obtain

$$h_{02} - h_{01} = \; U_2 c_{\theta 2} - U_1 c_{\theta 1} \tag{6}$$

This expression may also be written as

$$\Delta h_0 = \Delta(U c_\theta)$$

Let us now introduce the rothalpy as

$$I = h_0 - U c_\theta$$

By rearranging (6) one obtains

$$h_{01} - \; U_1 c_{\theta 1} = \; h_{02} - \; U_2 c_{\theta 2}$$

Or in other words,

$$I_1 = I_2$$

This proves the desired result.

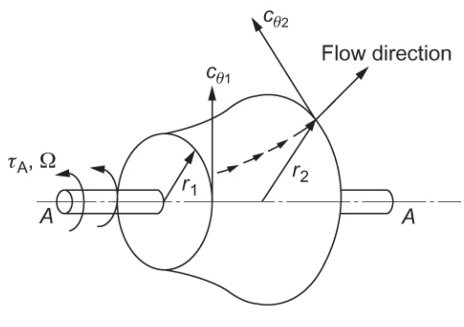

Figure 2: Flow along axi-symmetric stream surface inside a generalized turbomachine.

T3

Set $c_\theta \; = w_\theta + U$ and re-write the rothalpy in terms of the relative stagnation enthalpy and the blade speed, that is derive equation (1.21b) from (1.20b). How does $\Delta I \; = I_2 - I_1$, with I as defined by (1.21b) simplify over a rotor if the flow occurs on a cylindrical stream surface?

T4

Derive equation (1.39):

$$\frac{\dot{m}\sqrt{C_p T_0}}{A_n p_0} = \frac{\gamma}{\sqrt{\gamma - 1}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

T5 – Non-dimensional relationships –hydraulic case

Assume that the following functional relationships can be used to express the performance of a hydraulic turbomachine

$$gH = f_1\left(Q, N, D, \rho, \mu, e, \frac{l_1}{D}, \frac{l_2}{D}, \dots\right)$$

$$\eta = f_2\left(Q, N, D, \rho, \mu, e, \frac{l_1}{D}, \frac{l_2}{D}, \dots\right)$$

$$P = f_3\left(Q, N, D, \rho, \mu, e, \frac{l_1}{D}, \frac{l_2}{D}, \dots\right)$$

Based on these relations, derive the following expressions

$$\psi = \frac{gH}{(ND)^2} \approx f_4\left(\frac{Q}{ND^3}\right)$$

$$\eta = \frac{\rho Q g H}{P} \approx f_5\left(\frac{Q}{ND^3}\right)$$

$$\hat{P} = \frac{P}{\rho N^3 D^5} \approx f_6\left(\frac{Q}{ND^3}\right)$$

Clearly state the assumptions under which the above expressions are valid. See lecture 2.

T6 – Zweifel number

Derive the Zweifel number in terms of pitch (s), axial chord (b) and flow angles, that is derive equation (3.51).

T7 – Normal stage

Derive equation (4.14) for an axial flow turbine stage

$$\psi = 2(1 - R + \phi \tan \alpha_1)$$

Clearly state your assumptions. Based on this expression, how should you design a turbine stage with a high specific power output?

T8 – Turbine styles

Show that a turbine design with $R = 0.5$ results in symmetric blading, and that a turbine design with $R = 0$ results in equal relative inlet and outlet angles for the rotor.

T9 – Stage loading for compressor

Derive equation (5.17b) starting from the basic velocity triangles. Clearly state your assumptions.

T10 – Compressor losses

Which loss sources exist in compressors? Make a rough sketch of how they typically vary along the radius of a high speed compressor (see Lecture 5)

T11 – Radial equilibrium equation

Derive the equation (6.6a):

$$\frac{dh_0}{dr} - T\frac{ds}{dr} = c_x\frac{dc_x}{dr} + \frac{c_\theta}{r}\frac{d(rc_\theta)}{dr}$$

State your assumptions clearly.

T12 – Direct problem – constant α design

Derive equation (6.22):

$$\frac{c}{c_m} = \left(\frac{r}{r_m}\right)^{-\sin^2\alpha}$$

By starting from

$$\frac{dh_0}{dr} - T\frac{ds}{dr} = c_x\frac{dc_x}{dr} + \frac{c_\theta}{r}\frac{d(rc_\theta)}{dr}$$

Clearly state under which assumptions (6.22) is valid.

T13 – Performance of centrifugal compressors

Show that for a centrifugal compressor you can estimate the pressure ratio as:

$$\frac{p_{03}}{p_{01}} = (1 + (\gamma - 1)\eta_c\sigma(1 - \phi_2 \tan\beta_2')M_u^2)^{\frac{\gamma}{\gamma-1}}$$

Start from equation (1.18b).

T14 – Steam turbines

Define the isentropic degree of reaction R_s applicable to an impulse/reaction turbine (not the Curtis turbine case).

T15 – Steam turbines

Use the concept of density ratio, blade root stress, and flow coefficient to motivate why a typical steam turbine configuration may use a single flow HPT, a double flow IPT and a twin double LPT as shown in Figure 3.

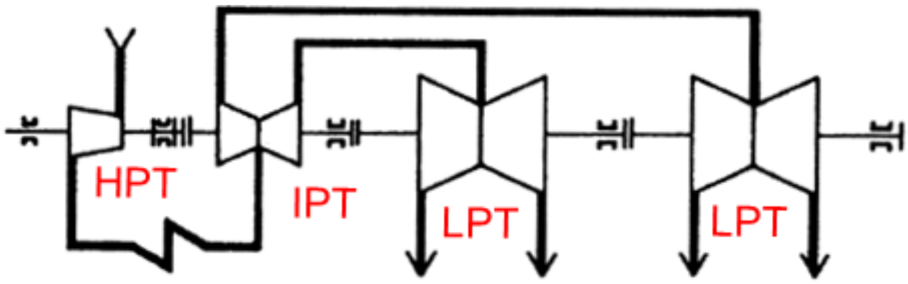

Figure 3: Typical configuration for a 600-800MW output: One single flow HPT, one double flow IPT and two double flow LPT.

In other words, why does the plant layout in Figure 3 make sense if you take into consideration design constraints arising from thermodynamics, solid mechanics and fluid dynamics?

T16 – Wind turbines

Derive the Betz limit by establishing an expression for the power coefficient C_p (see Lecture 11) and differentiate on it with respect to the velocity ratio c_{x3}/c_{x1} (you may of course also follow the book which differentiate with respect to $\bar{a}$). You may use the Rankine-Froude theorem without deriving it. What is the interpretation of the limit that you establish?