

L11: a summary

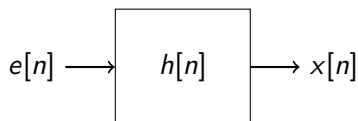
Tomas McKelvey

Signal Processing Group
Department of Signals and Systems
Chalmers University of Technology, Sweden



Some useful relations

- 1 If $r_x[n] \leftrightarrow P_x(z)$ then $P_x(e^{j\omega}) = P_x(z)|_{z=e^{j\omega}}$.
- 2 The system



$$\Rightarrow \begin{cases} X(z) = H(z)E(z) \\ P_x(e^{j\omega}) = |H(e^{j\omega})|^2 P_e(e^{j\omega}) \end{cases} \quad (\text{assuming that they exist})$$

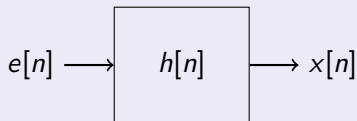
- 3 Rules for z-transform

$$\delta[n] \leftrightarrow 1$$

$$r_x[n-1] \leftrightarrow z^{-1}P_x(z)$$

$$r_x[n-m] \leftrightarrow z^{-m}P_x(z)$$

A stochastic model for $x[n]$



where $e[n]$ is WSS, white noise,

$$\Rightarrow \begin{cases} \mathbb{E}\{e[n]\} = 0 \\ r_e[k] = \mathbb{E}\{e[n]e[n-k]\} = \delta[k]\sigma_e^2 = \begin{cases} \sigma_e^2 & \text{if } k = 0 \\ 0 & \text{otherwise.} \end{cases} \end{cases}$$

- It follows that

$$P_e(e^{j\omega}) = DTFT\{r_e[k]\} = \sigma_e^2 DTFT\{\delta[k]\} = \sigma_e^2.$$

- The filter $H(z)$ is used to shape the autocorrelation/spectrum of the process $x[n]$, $P_x(e^{j\omega}) = |H(e^{j\omega})|^2 \sigma_e^2$.

AR(p) model

$$x[n] + a_1x[n-1] + \cdots + a_px[n-p] = e[n]$$

MA(q) model

$$x[n] = e[n] + b_1e[n-1] + \cdots + b_qe[n-q]$$

ARMA(p, q) model

$$x[n] + a_1x[n-1] + \cdots + a_px[n-p] = e[n] + b_1e[n-1] + \cdots + b_qe[n-q]$$