L11: a summary

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1. If $r_x[n] \leftrightarrow P_x(z)$ then $P_x(e^{j\omega}) = P_x(z)\big|_{z=e^{j\omega}}$.
2. The system

$$e[n] \longrightarrow \boxed{\; h[n] \;} \longrightarrow x[n]$$

$$\Rightarrow \begin{cases} X(z) = H(z)E(z) \\ P_x(e^{j\omega}) = \left|H(e^{j\omega})\right|^2 P_e(e^{j\omega}) \end{cases} \qquad \text{(assuming that they exist)}$$

3. Rules for z-transform

$$\delta[n] \leftrightarrow 1$$
$$r_x[n-1] \leftrightarrow z^{-1} P_x(z)$$
$$r_x[n-m] \leftrightarrow z^{-m} P_x(z)$$

Signal models: AR, MA and ARMA processes

A stochastic model for $x[n]$

$$e[n] \longrightarrow \boxed{h[n]} \longrightarrow x[n]$$

where $e[n]$ is WSS, white noise,

$$\Rightarrow \begin{cases} \mathbb{E}\{e[n]\} = 0 \\ r_e[k] = \mathbb{E}\{e[n]e[n-k]\} = \delta[k]\sigma_e^2 = \begin{cases} \sigma_e^2 & \text{if } k = 0 \\ 0 & \text{otherwise.} \end{cases} \end{cases}$$

- It follows that $P_e(e^{j\omega}) = DTFT\{r_e[k]\} = \sigma_e^2 DTFT\{\delta[k]\} = \sigma_e^2$.
- The filter $H(z)$ is used to shape the autocorrelation/spectrum of the process $x[n]$, $P_x(e^{j\omega}) = |H(e^{j\omega})|^2\sigma_e^2$.

The models

AR(p) model

$$x[n] + a_1 x[n-1] + \cdots + a_p x[n-p] = e[n]$$

MA(q) model

$$x[n] = e[n] + b_1 e[n-1] + \cdots + b_q e[n-q]$$

ARMA(p, q) model

$$x[n]+a_1 x[n-1]+\cdots+a_p x[n-p] = e[n]+b_1 e[n-1]+\cdots+b_q e[n-q]$$