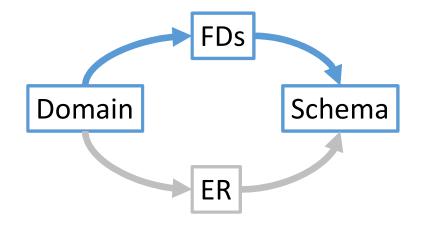
# TDA357/DIT621 – Databases

Lecture 6 – Design using Functional Dependencies and normal forms Jonas Duregård

# Another high level design approach

- This week we will look at functional dependencies (FDs) and normal forms
- This is an alternative (and to some degree complementary) approach to ER that we studied last week
  - We start in a domain description and end in a database schema
- A single lecture, and Friday exercises as usual



# Normalisation in a nutshell

- Extract a bunch of formal statements from the domain description
- Compute a normalised database schema from the formal statements



- Highly systematic, almost mechanical process
- By a carefully constructed normalization algorithm, the normal form the schema ends up in will satisfy some important properties

# Functional Dependencies (FDs)

- A functional dependency is written as <set of attributes>  $\rightarrow$  <attribute>
- Example: room time→course ← Do not confuse with references!
- Pronounced "room and time determines course"
- It is a statement that can be true or false
- A few ways of understanding the meaning of the statement above:
  - If we know room and a time, we can uniquely determine course
  - There can be at most one course value for each (room,time)-pair
  - There exists a partial function *f* that takes a room and a time and yields a course
- In a domain it might have said something like "courses can book rooms at any free times"

# Three ways we can use functional dependencies

- Check if they hold for a specific data set
- Check if a design ensures they hold for all data sets
- Express desired properties of a design
- I will explain each of these in turn

# Functional dependency as a property of data

- One way of formally defining functional dependency x<sub>1</sub> x<sub>2</sub> ... → y : For R(y x<sub>1</sub> x<sub>2</sub> ...), if two rows agree on x<sub>1</sub>, x<sub>2</sub> ... they must also agree on y
- In other words, there can not exist two rows where the left hand side attributes are the same, but the right hand side attribute differs
  - "X  $\rightarrow$  y = rows that agree on X must agree on y"

Two rows "agreeing on x" just means the x-column(s) have the same value

#### Table: Bookings

• Which FDs hold for this data?

courseCode	name	day	timeslot	room	seats
TDA357	Databases	Tuesday	0	GD	236
TDA357	Databases	Tuesday	1	GD	236
ERE033	Reglerteknik	Tuesday	0	HB4	224
ERE033	Reglerteknik	Friday	0	GD	236

courseCode  $\rightarrow$  name?

- Yes! (TDA357 maps to Databases, ERE033 to Reglerteknik)
- day  $\rightarrow$  timeslot?
  - No! (Tuesday maps to both 0 and 1)
- day timeslot room  $\rightarrow$  courseCode?

LHS = Left hand side (of arrow) RHS = Right hand side

• Yes! (There are no rows where all three LHS columns match)

seats  $\rightarrow$  room?

- Yes! 236 for GD, 224 for HB4,
- This might not be intentional given what we know of the domain...

### FDs as a properties of designs

- Knowing what it means for an FD to hold for a data set, we can determine if a design (schema) guarantees that it holds for all valid data sets
- Example: Does the schema below guarantee that ...

 $code \rightarrow cname?$ 

• Yes! (by primary key constraint in courses)

cname  $\rightarrow$  code?

• No! (Counterexample: any two courses with the same name)

 $\mathsf{code} \rightarrow \mathsf{email}$ 

• Yes! (teacher is just another name for email)

 $code \rightarrow tname$ 

• Yes! (by primary key + reference)

Teachers(<u>email</u>, tname) Courses(<u>code</u>, cname, teacher) teacher -> Teachers.email

#### Bookings(<u>courseCode</u>, name, <u>day</u>, <u>timeslot</u>, room, seats) (day, timeslot, room) UNIQUE

- Does the schema above guarantee ...
- day timeslot room → courseCode
  - Yes (through UNIQUE constraint)
- day timeslot room coursCode  $\rightarrow$  seats
  - Yes (through primary key and/or UNIQUE)
- room  $\rightarrow$  seats
  - No 🛞
- courseCode  $\rightarrow$  name
  - No 🛞

courseCode	name	day	timeslot	room	seats	
CC1	N1	Tuesday	0	R1	0	
CC1	N2	Tuesday	1	R1	1	
Different timeslot, so no key violation						

Counterexample of room  $\rightarrow$  seats and courseCode  $\rightarrow$  name

# FDs as intention for designs

- Since we can verify that an FD holds for a schema, we can also use them to specify desired properties of our schema
- This is what makes FDs a design tool
- A sentence like "every course has a teacher" can be modelled as the FD course → teacher (or whatever attributes we use)
  - If this FD does not hold for our design, maybe the design is bad?

## Formal properties of FDs

Warning: Things may get slightly mathsy from this point

- FDs have lots of interesting mathematical properties
- I will explain some of the more useful ones:
  - Transitivity
  - Augmentation
  - Reflexivity
- These three are commonly referred to as Armstrongs axioms and they can be formulated in a few different but equivalent ways\*

\*But the way I formulate them is -of course- the best way

# Side note: Single/Multiple FDs

Notation: I use lowercase x/y/z for single attributes and uppercase X/Y/Z for attribute sets

- It is common to have multiple attributes on the right hand side of FDs x y z → a b c
- This means exactly the same as these three FDs:
  - $x y z \rightarrow a$ x y z \rightarrow b x y z \rightarrow c
- I find it most useful to think of the first as a convenient way of writing multiple FDs, rather than thinking of it as single FD with multiple attributes
- It is <u>not</u> the same with the left hand side!  $x y \rightarrow a$  does not mean  $x \rightarrow a$ !

Recall:  $X \rightarrow y = rows$  that agree on X must agree on y

# Transitivity of functional dependencies

- Functional dependency is a transitive relation
  - This means that if  $X \rightarrow Y$  and  $Y \rightarrow Z$  then  $X \rightarrow Z$
- Note that Y is an attribute set here, so  $X \rightarrow Y$  may be multiple FDs with the same LHS
- Proof sketch: Look at any rows that agree on X. Since X → Y, they must also agree on Y, and since Y → Z they must further agree on Z. Thus X → Z.

Recall:  $X \rightarrow y = rows$  that agree on X must agree on y

#### Augmentation

- If  $x_1 x_2 \dots \rightarrow y$ , then for all  $z: z x_1 x_2 \dots \rightarrow y$
- Intuitively: You can add any attributes you want to the LHS of a valid FD and still get a valid FD
  - Think: "knowing an extra attribute never prevent us from finding y"
- Proof sketch: Since all rows that agree on the xs must agree on y, then particularly all rows that agree on z as well as the xs must do so.

Recall:  $X \rightarrow y = rows$  that agree on X must agree on y

#### Reflexivity and trivial FDs

- For all x: x → x
   (x determines itself)
- By augmentation,  $X \rightarrow y$  whenever  $y \in X$ 
  - Example: a b c  $\rightarrow$  b
  - We call these depencies trivial
  - Rule of thumb: Ignore trivial dependencies
- Proof sketch: Any values that agree on x will agree on x 🙂

#### Example: Deriving functional dependencies

• For any attributes x, y, z, w, and q: x  $\rightarrow$  y, z  $\rightarrow$  w, and y w  $\rightarrow$  q implies x z  $\rightarrow$  q  $\begin{array}{c} x \rightarrow y \\ z \rightarrow w \\ \frac{y w \rightarrow q}{x z \rightarrow q} \end{array}$ 

#### • Proof:

- $x \rightarrow y$  implies  $x z \rightarrow y$  (by augmentation)
- $z \rightarrow w$  implies  $x z \rightarrow w$  (by augmentation)
- x z  $\rightarrow$  y w and y w  $\rightarrow$  q implies x z  $\rightarrow$  q (by transitivity)
- Note that in the third step we merge x z → y and x z → w into x z → y w (See slide on Single/Multiple FDs)

#### Minimal basis

- The minimal basis F<sup>-</sup> of a set of functional dependencies F is a set equivalent to F but with the following conditions:
  - F- has no trivial dependenices
  - No dependency in F<sup>-</sup> follow from other dependencies in F<sup>-</sup> through transitivity or augmentation
- Used for a lot of algorithms and to express a set of FDs in a compact form

# Minimal basis

- Suppose we are given this set of FDs (5 total), what is a minimal basis?
   a → b
   b → c
   a d → b c d
- a d  $\rightarrow$  d is removed because it is trivial
- a d  $\rightarrow$  b is removed because it is implied by a  $\rightarrow$  b (augmentation)
- a d → c is removed because it is implied by a → b and b → c (transitivity and augmentation)
- Final set:  $a \rightarrow b$ ,  $b \rightarrow c$

#### Transitive closure

- The transitive closure X<sup>+</sup> of a set of attributes X, is the set of attributes that can be functionally determined by X
- In other words  $X^+$  = All attributes y such that  $X \rightarrow y$ 
  - Includes ALL derived functional dependencies
  - Includes trivial dependencies
  - X<sup>+</sup> is closed in the sense that any FD from attributes in X<sup>+</sup> lead back to X<sup>+</sup>
- Can be computed by a simple algorithm from any set of FDs:
  - Start with X<sup>+</sup> = X (an under-approximation)
  - Repeat until done: For any FD Y  $\rightarrow$  z such that Y  $\subseteq$  X<sup>+</sup>, add z to X<sup>+</sup>

#### Transitive closure, example

- Given these FDs, compute the closure {x,z}<sup>+</sup>
- Initially we know  $\{x,z\} \subseteq \{x,z\}^+$  (from trivial FDs)
- Add y because  $x \rightarrow y$  and  $\{x\} \subseteq \{x,z\}^+$
- Add w because  $z \rightarrow w$  and  $\{x\} \subseteq \{x,z\}^+$
- Add q because y w  $\rightarrow$  q and {y,w}  $\subseteq$  {x,z}<sup>+</sup>
- No more FDs add attributes, so {x,z}<sup>+</sup> = {x,z,y,w,q} is our result
- This proves all these non-trivial FDs:

 $x z \rightarrow y$   $x z \rightarrow w$   $x z \rightarrow q$ 

$$\begin{array}{l} x \rightarrow y \\ y w \rightarrow q \\ z \rightarrow w \\ q \rightarrow x \\ r \rightarrow s \end{array}$$

 $\{x, z, y\} \subseteq \{x, z\}^+$  $\{x, z, y, w\} \subseteq \{x, z\}^+$  $\{x, z, y, w, q\} \subseteq \{x, z\}^+$ 

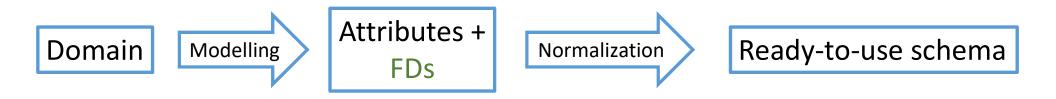
### Keys and superkeys

- We can define the property of being a key of a relation using FDs
- Intuitively: A set of attributes is a *superkey* if it determines all other attributes
- Formally: The attribute set X is a *superkey* of R if X<sup>+</sup> contains all attributes of R
- X is a (minimal) key if removing any attribute from X makes it a non-superkey
  - Saying only "key" usually means minimal key
  - Each superkey is a superset of at least one minimal key
  - Each key is a superkey (but not the other way around)
  - Adding any attribute to a superkey makes a new superkey

# Summary so far

- An FD X  $\rightarrow$  y means any rows that agree on X also agree on Y
- We can extend a set of FDs with additional implied FDs using transitivity, augmentation, and reflexivity
- Conversely, we can reduce a set of FDs to a minimal basis by removing all implied FDs
- The closure X<sup>+</sup> is the set of all attributes that can be determined by X
- A superkey is a set of attributes that determine all other, keys are minimal superkeys
- To find a key: Start with all attributes (a superkey) and remove attributes until it is a key – finding all keys is more work though

### Normal forms and normalization



# Normal forms and normalization

- Normal form is a very important concept in database design
- Identify all the attributes in the domain and place them in one big relation D(x, y, z, ...), collect FDS, then normalize D to get your design
- Normalizing is a recursive procedure, to normalize relation R:
  - Check if R is already a normal form, if it is we are done
  - Otherwise decompose R into relations R<sub>1</sub> and R<sub>2</sub> and normalize both
- Note: A normal form is not the same as a canonical form, there may be multiple normal forms derived from the same initial domain

#### BCNF, the Boyce-Codd Normal Form

Arguably the most well known normal form

# BCNF Normalisation algorithm

To normalize relation R:

This FD is referred to as a BCNF-violation

- Find a non-trivial FD X  $\rightarrow$  y such that X<sup>+</sup>  $\neq$  R (X is not a superkey)
- If there is no such FD you are done
- Otherwise decompose R into  $R_1(X^+)$  and  $R_2(X \cup (R X^+))$  and normalize them

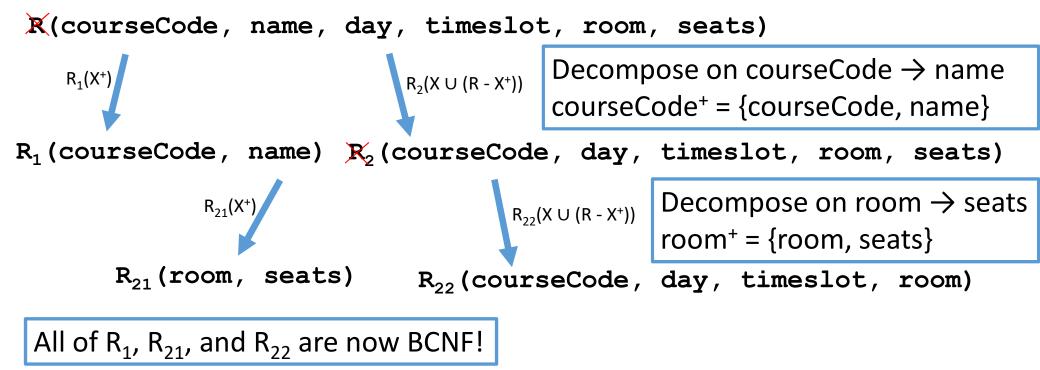
Note: R is replaced by  $R_1$  and  $R_2$  (so R is not present in the final schema)

1. Find violation
2. Decompose
3. Repeat

Example

 $courseCode \rightarrow name$ room  $\rightarrow$  seats day timeslot courseCode  $\rightarrow$  room day timeslot room  $\rightarrow$  courseCode

Normalise this relation using the FDs above:



#### Wait, why not split on day timeslot course $\rightarrow$ room?

 $R_{22}$ (courseCode, day, timeslot, room) day timeslot courseCode → room day timeslot room → courseCode

Recall: Find a non-trivial FD X  $\rightarrow$  y such that  $X^+ \neq R$  (X is no superkey)

{day, timeslot, courseCode}<sup>+</sup> = { day, timeslot, courseCode, room} =  $R_{22}$ 

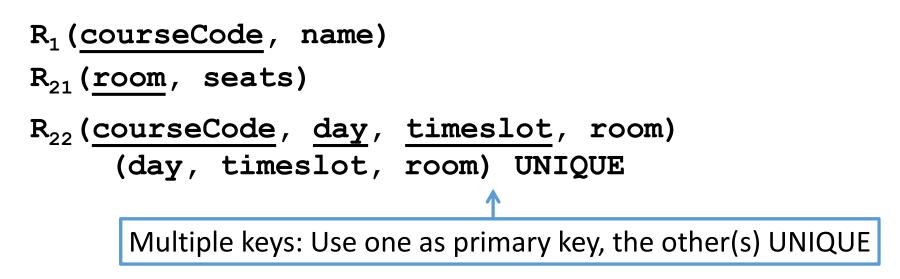
{day, timeslot, room}<sup>+</sup> = { day, timeslot, room, courseCode} = R<sub>22</sub>

Both {day, timeslot, courseCode} and {day, timeslot, room} are keys!

### What about keys?

```
courseCode \rightarrow name
room \rightarrow seats
day timeslot courseCode \rightarrow room
day timeslot room \rightarrow courseCode
```

- Keys can be determined using FDs (and closures) after decomposing
- Much of it is already done as part of the algorithm (we found two keys for  $R_{22}$  for instance)



#### What about references?

• In this case it's fairly easy to see that these are sensible references:

 $\begin{array}{l} R_1 \left( \underline{\text{courseCode}} \,, \, \text{name} \right) \\ R_{21} \left( \underline{\text{room}} \,, \, \text{seats} \right) \\ R_{22} \left( \underline{\text{courseCode}} \,, \, \underline{\text{day}} \,, \, \underline{\text{timeslot}} \,, \, \text{room} \right) \\ \left( \text{day} \,, \, \text{timeslot} \,, \, \text{room} \right) \, \text{UNIQUE} \\ \text{courseCode} \, -> \, R_1 \,. \, \text{courseCode} \\ \text{room} \, -> \, R_{21} \,. \, \text{room} \end{array}$ 

- General pattern: When decomposing R, add a reference X -> R<sub>1</sub>.X to R<sub>2</sub>
- This will not always work, particularly if  $R_1$  or  $R_2$  is later decomposed  $\otimes$

#### Decomposition of data

#### No redundancy, no anomalies 😳

#### **Table: Bookings**

courseCode	name	day	timeslot	room	seats
TDA357	Databases	Tuesday	0	GD	236
TDA357	Databases	Tuesday	1	GD	236
ERE033	Reglerteknik	Tuesday	0	HB4	224
ERE033	Reglerteknik	Friday	0	GD	236

Table: R<sub>1</sub> (Courses)

courseCode	name
TDA357	Databases
ERE033	Reglerteknik

Table: R <sub>22</sub> (Bookings)					
courseCode	day	timeslot	room		
TDA357	Tuesday	0	GD		
TDA357	Tuesday	1	GD		
ERE033	Tuesday	0	HB4		
ERE033	Friday	0	GD		

Table R <sub>21</sub> (Rooms)				
room	seats			
HB4	224			
GD	236			

#### Lossless join

#### • Note that if we join along the references, we get the original table

Table: R <sub>1</sub> (Courses)			
courseCode	name		
TDA357	Databases		
ERE033	Reglerteknik		

Table: R<sub>22</sub> (Bookings)

courseCode	day	timeslot	room
TDA357	Tuesday	0	GD
TDA357	Tuesday	1	GD
ERE033	Tuesday	0	HB4
ERE033	Friday	0	GD

Table R <sub>21</sub>	Table R <sub>21</sub> (Rooms)				
room	seats				
HB4	224				
GD	236				

#### Joins ON (courseCode) and ON (room)

#### Query: R1 NATURAL JOIN R22 NATURAL JOIN R21

 Means we did not loose any data in the decomposition

	courseCode	name	day	timeslot	room	seats
1	TDA357	Databases	Tuesday	0	GD	236
	TDA357	Databases	Tuesday	1	GD	236
	ERE033	Reglerteknik	Tuesday	0	HB4	224
	ERE033	Reglerteknik	Friday	0	GD	236

# Finding all FDs

- Consider this simple situation with four attributes R(x,y,z,w) and two functional dependencies: x → z and y z → w
- When normalizing R it may be important to know that there is another FD that can be derived from these: x y → w
- In principle, you should consider all non-trivial derived FDs but sometimes this a large set and it is easy to miss FDs
  - Essentially you have to consider every LHS and compute closures
  - There are some clever tricks you can use, but we will not have time for those today

# A flaw of BCNF

- Same example as before: R(x,y,z,w) where  $x \rightarrow z$  and  $y z \rightarrow w$
- If we decompose on  $x \rightarrow z$  ({x}<sup>+</sup> = {x,z}) we get
  - $R_1(\underline{x},z)$  {x} is the only key
  - $R_2(\underline{x},\underline{y},w)$  {x,y} is the only key
- Both of these relations are in BCNF w.r.t. the given FDs
- But now y z  $\rightarrow$  w is not guaranteed by the schema  $\otimes$
- There is a weaker normal form called third normal form (3NF) that does not have this problem, but it has other problems instead...

Because  $x y \rightarrow w$ 

• There is no "silver bullet" for design work

# Yet another issue with BCNF

• This relation has no non-trivial functional dependencies, so is in BCNF: Table: Courses

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Aarne
Reglerteknik	RTB 1	Author1	Teacher3
Reglerteknik	RTB 2	Author2	Teacher3

Deletion anomaly: Deleting all course books also deletes all teachers

Update anomaly: Changing some value can cause inconsistencies

- The domain said something like "each course has a number of teachers and a number of books with one or more authors" (no FDs at all!)
- The data above says Databases has one book and two teachers, and Reglerteknik has two books and one teacher
- Clearly there is redundancy here, and potential for anomalies

# Looks like we need another normal form!

- This one is called the fourth normal form (4NF)
- Since the problematic table had no FDs at all, this form will need some additional source of facts
- We call these facts multivalued dependencies (MVDs)\*
- We write  $x_1 x_2 x_3 \dots \twoheadrightarrow y_1 y_2 y_3 \dots$ 
  - Note that both sides are sets of values and we can <u>not</u> split the RHS

\*The term multivalued dependency is really quite unfortunate, but it is what it is

### Multivalued dependencies, informally

- This means that if we fix a course value, the teacher value is independent from all other values (author and book)

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Aarne
Reglerteknik	RTB 1	Author1	Teacher3
Reglerteknik	RTB 2	Author2	Teacher3

• This is exactly the same as saying course  $\rightarrow$  book author

### Multivalued dependencies, formally

• The claim that X->>Y holds for relation R means:

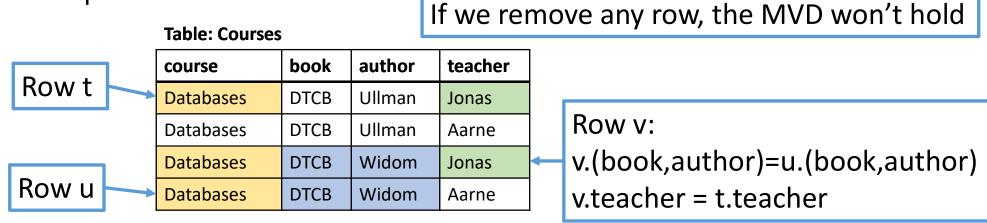
For every pair of rows row t and u in R that agree on X we can find a row v s.t:

v agrees with both t and u on X

v agrees with t on Y

v agrees with u on R - X - Y (all attributes not in the MVD)

• Example: course ---> teacher



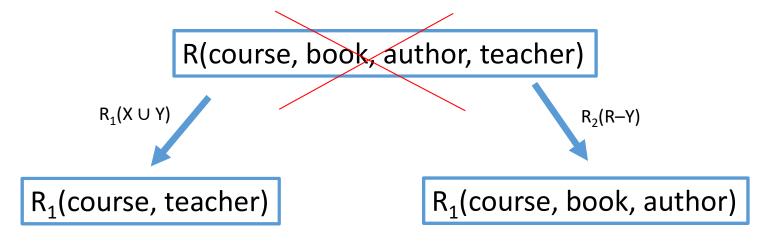
### Verifying MVDs on data is hard

- To check if an FD holds: Just group values up by the LHS and check that all rows in each group have the same value for the RHS
- To check if an MVD holds: Check every individual pair of values with identical LHS and search for a row with correct values
- I find a more intuitive way of thinking is this: For X→Y, every X needs to have every possible combination of Y and other attributes (R-X-Y)
  - Essentially the rows for a given X must be a cartesian product!
  - If teacher Jonas occurs with one book/autor, it must occur with all book/author combinations for that course
  - This is what makes (book, author) independent from teacher

### Fourth normal form

- For a relation R to be in fourth normal:
  - R must be in BCNF
  - For all non-trivial MVDs X->>Y on R, X is a superkey of R
- If X->>Y and X is not a superkey, we say X->>Y is a 4NF violation
- To normalize: Find a violation X->>Y and break R into
  - $R_1(X \cup Y)$  ("every attribute in the MVD")
  - R<sub>2</sub>(R Y) ("LHS and every attribute <u>not</u> in the MVD")
  - Then normalize both R<sub>1</sub> and R<sub>2</sub>

### 4NF normalisation



### Normalising the data

**Table: Courses** 

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Aarne
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Aarne
Reglerteknik	RTB 1	AuthorX	TeacherX
Reglerteknik	RTB 2	AuthorX	TeacherX

# Exercise: Find another MVD here?

Table: R<sub>1</sub> (a.k.a. CourseTeacher)

course	teacher
Databases	Jonas
Databases	Aarne
Reglerteknik	Teacher3

#### Table: R2 (a.k.a. CourseBooks)

course	book	author
Databases	DTCB	Ullman
Databases	DTCB	Widom
Reglerteknik	RTB 1	AuthorX
Reglerteknik	RTB 2	AuthorX

### Lossless join

#### Note that if we join the two tables using course ...

#### Table: R<sub>1</sub> (a.k.a. CourseTeacher)

course	teacher
Databases	Jonas
Databases	Aarne
Reglerteknik	TeacherX

#### Table: R2 (a.k.a. CourseBooks)

course	book	author
Databases	DTCB	Ullman
Databases	DTCB	Widom
Reglerteknik	RTB 1	AuthorX
Reglerteknik	RTB 2	AuthorX

### NATURAL JOIN 🖌

## We get the original table back!

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Aarne
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Aarne
Reglerteknik	RTB 1	AuthorX	TeacherX
Reglerteknik	RTB 2	AuthorX	TeacherX

Sanity check: We did not loose any information

### Functional dependencies vs. ER-design

- FDs can find some things that ER can not find
- ER can find a lot of things that FDs can not find
  - Most many-to-many relationships can not be expressed using FDs
  - Sentences like "students can register for courses" do not express any FDs (but possibly some MVDs?)
- The two approaches complement eachother, and confirm eachother (or sometimes contradict eachother which may indicate a problem)
- So doing both an ER-design and a FD analysis may be useful
  - This is what you will do in Task 2

### Practical use of FDs combined with ER

- FDs can be used to verify the correctness of an ER-design
  - Is the result in BCNF w.r.t. the dependencies you have identified?
  - Are the primary keys you identified sensible from your FDs?
  - If not there may be an error in your ER-translation or your understanding/modelling of the domain
- Sometimes FDs can be used to patch things up in your ER-design, particularly they are useful for finding secondary keys (UNIQUE constraints)
  - Every (minimal) key of each relation should be either the primary key or unique

### Finding functional dependencies

- Determine all attributes
- Discover FD's either by looking at each attribute and ask "what do i need to know to determine this?" or by looking at each fact in the domain description and asking "does this express a dependency?"
- You can find multiple FDs determining the same attribute

### Mining attributes (and FDs) from ER-design

- If you already have an ER-design, that may help you determine a useful set of attributes
- Looking at the relational schema is less helpful, because it contains multiple attributes that have different names but are conceptually the same (i.e. because of references)
- You can also extract some FDs by studying the diagram/schema, but that sort of misses the point of finding them since you will never find any FDs that can improve your design
  - We want to find FDs that express things our ER-design is missing
  - We should look for FDs in the domain description

### Other normal forms

- There is a whole little hierarchy of normal forms Higly simplified:
- 1NF: basically means "only has actual tables"
- 2NF: 1NF + has valid primary key
- 3NF: 2NF + no FDs between attributes not in keys
- BCNF a.k.a. 3½NF: 3NF + attributes depend only on keys
- 4NF: 3NF + No violating MVDs
- 5NF, 6NF, DK/NF ...: Outside the scope of this course
- I expect you to know how to normalize to BCNF and 4NF

TF	
21F	
STF	
BENF	

### I somehow doubt I will actually reach this slide

- So who cares what I write here?
- If a tree falls in the forest and nobody hears, does it make a sound? If a slide is the 50<sup>th</sup> slide of a 40 slide lecture, does it even exist?
- Well, there's the course page I guess, but who uses Canvas, am I right?
- Live and learn people: Don't make slides late at night or things are bound to get silly towards the end.