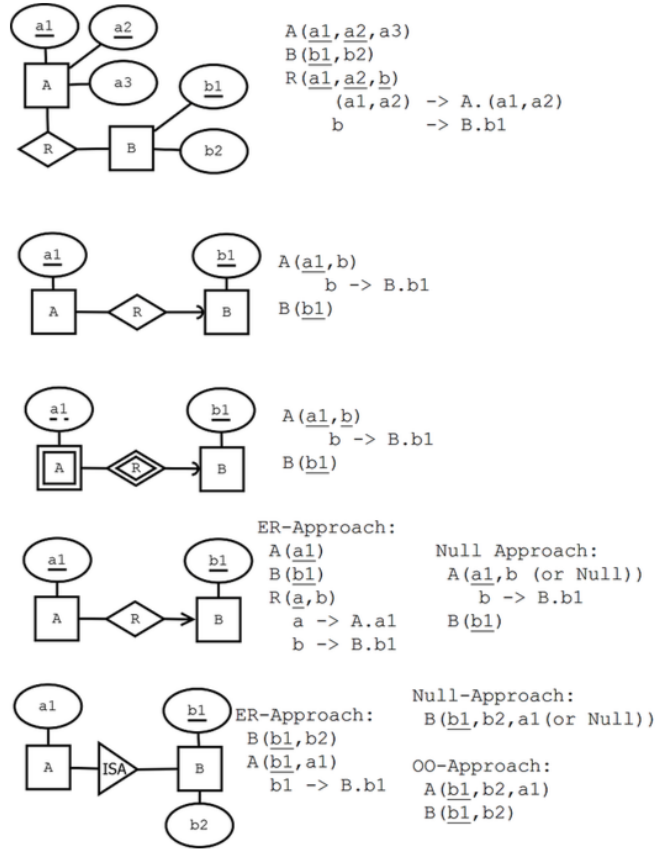


## E-R diagrams and database schemas



## Functional dependencies

**Definition** (tuple, attribute, value). A **tuple** has the form

$$\{A_1 = v_1, \dots, A_n = v_n\}$$

where  $A_1, \dots, A_n$  are **attributes** and  $v_1, \dots, v_n$  are their **values**.

**Definition** (signature, relation). The **signature** of a tuple,  $S$ , is the set of all its attributes,  $\{A_1, \dots, A_n\}$ . A **relation**  $R$  of signature  $S$  is a set of tuples with signature  $S$ . But we will sometimes also say "relation" when we mean the signature itself.

**Definition** (projection). If  $t$  is a tuple of a relation with signature  $S$ , the **projection**  $t.A_i$  computes to the value  $v_i$ .

**Definition** (simultaneous projection). If  $X$  is a set of attributes  $\{B_1, \dots, B_m\} \subseteq S$  and  $t$  is a tuple of a relation with signature  $S$ , we can form a simultaneous projection,

$$t.X = \{B_1 = t.B_1, \dots, B_m = t.B_m\}$$

**Definition** (functional dependency, FD). Assume  $X$  is a set of attributes and  $A$  an attribute, all belonging to a signature  $S$ . Then  $A$  is **functionally dependent** on  $X$  in the relation  $R$ , written  $X \rightarrow A$ , if

- for all tuples  $t, u$  in  $R$ , if  $t.X = u.X$  then  $t.A = u.A$ .

If  $Y$  is a set of attributes, we write  $X \rightarrow Y$  to mean that  $X \rightarrow A$  for every  $A$  in  $Y$ .

**Definition** (multivalued dependency, MVD). Let  $X, Y, Z$  be disjoint subsets of a signature  $S$  such that  $S = X \cup Y \cup Z$ . Then  $Y$  has a **multivalued dependency** on  $X$  in  $R$ , written  $X \twoheadrightarrow Y$ , if

- for all tuples  $t, u$  in  $R$ , if  $t.X = u.X$  then there is a tuple  $v$  in  $R$  such that
  - $v.X = t.X$
  - $v.Y = t.Y$
  - $v.Z = u.Z$

**Definition.** An attribute  $A$  **follows** from a set of attributes  $Y$ , if there is an FD  $X \rightarrow A$  such that  $X \subseteq Y$ .

**Definition** (closure of a set of attributes under FDs). The **closure** of a set of attributes  $X \subseteq S$  under a set FD of functional dependencies, denoted  $X^+$ , is the set of those attributes that follow from  $X$ .

**Definition** (trivial functional dependencies). An FD  $X \rightarrow A$  is **trivial**, if  $A \in X$ .

**Definition** (superkey, key). A set of attributes  $X \subseteq S$  is a **superkey** of  $S$ , if  $S \subseteq X^+$ .

A set of attributes  $X \subseteq S$  is a **key** of  $S$  if

- $X$  is a superkey of  $S$
- no proper subset of  $X$  is a superkey of  $S$

**Definition** (Boyce-Codd Normal Form, BCNF violation). A functional dependency  $X \rightarrow A$  **violates BCNF** if

- $X$  is not a superkey
- the dependency is not trivial

A relation is in **Boyce-Codd Normal Form** (BCNF) if it has no BCNF violations.

**Definition** (prime). An attribute  $A$  is prime if it belongs to some key.

**Definition** (Third Normal Form, 3NF violation). A functional dependency  $X \rightarrow A$  **violates 3NF** if

- $X$  is not a superkey
- the dependency is not trivial
- $A$  is not prime

**Definition** (trivial multivalued dependency). A multivalued dependency  $X \twoheadrightarrow A$  is trivial if  $Y \subseteq X$  or  $X \cup Y = S$ .

**Definition** (Fourth Normal Form, 4NF violation). A multivalued dependency  $X \twoheadrightarrow A$  **violates 4NF** if

- $X$  is not a superkey
- the MVD is not trivial.

**Algorithm** (BCNF decomposition). Consider a relation  $R$  with signature  $S$  and a set  $F$  of functional dependencies.  $R$  can be brought to BCNF by the following steps:

1. If  $R$  has no BCNF violations, return  $R$
2. If  $R$  has a violating functional dependency  $X \rightarrow A$ , decompose  $R$  to two relations
  - $R_1$  with signature  $X^+$
  - $R_2$  with signature  $X \cup (S - X^+)$
3. Apply the above steps to  $R_1$  and  $R_2$  with functional dependencies projected to the attributes contained in each of them.

**Algorithm** (4NF decomposition). Consider a relation  $R$  with signature  $S$  and a set  $M$  of multivalued dependencies.  $R$  can be brought to 4NF by the following steps:

1. If  $R$  has no 4NF violations, return  $R$
2. If  $R$  has a violating multivalued dependency  $X \twoheadrightarrow Y$ , decompose  $R$  to two relations
  - $R_1$  with signature  $X \cup Y$
  - $R_2$  with signature  $S - Y$
3. Apply the above steps to  $R_1$  and  $R_2$

**Concept** (minimal basis of a set of functional dependencies; not a rigorous definition). A **minimal basis** of a set  $F$  of functional dependencies is a set  $F^-$  that implies all dependencies in  $F$ . It is obtained by first weakening the left hand sides and then dropping out dependencies that follow by transitivity. Weakening an LHS in  $X \rightarrow A$  means finding a minimal subset of  $X$  such that  $A$  can still be derived from  $F^-$ .

**Algorithm** (3NF decomposition). Consider a relation  $R$  with a set  $F$  of functional dependencies.

1. If  $R$  has no 3NF violations, return  $R$ .
2. If  $R$  has 3NF violations,
  - compute a minimal basis of  $F^-$  of  $F$
  - group  $F^-$  by the left hand side, i.e. so that all dependencies  $X \rightarrow A$  are grouped together
  - for each of the groups, return the schema  $XA_1 \dots A_n$  with the common LHS and all the RHSs
  - if one of the schemas contains a key of  $R$ , these groups are enough; otherwise, add a schema containing just some key

## Relational algebra

relation ::=	
relname	<b>name of relation (can be used alone)</b>
$\sigma_{\text{condition}}$ relation	<b>selection (sigma) WHERE</b>
$\pi_{\text{projection+}}$ relation	<b>projection (pi) SELECT</b>
$\rho_{\text{relname (attribute+)}}$ ? relation	<b>renaming (rho) AS</b>
$\gamma_{\text{attribute*,aggregationexp+}}$ relation	
$\tau_{\text{expression+}}$ relation	<b>grouping (gamma) GROUP BY, HAVING</b>
$\delta$ relation	<b>sorting (tau) ORDER BY</b>
relation $\times$ relation	<b>removing duplicates (delta) DISTINCT</b>
relation $\cup$ relation	<b>cartesian product FROM, CROSS JOIN</b>
relation $\cap$ relation	<b>union UNION</b>
relation $-$ relation	<b>intersection INTERSECT</b>
relation $\bowtie$ relation	<b>difference EXCEPT</b>
relation $\bowtie_{\text{condition}}$ relation	<b>NATURAL JOIN</b>
relation $\bowtie_{\text{attribute+}}$ relation	<b>theta join JOIN ON</b>
relation $\bowtie_{\text{attribute+}}^o$ relation	<b>INNER JOIN</b>
relation $\bowtie_{\text{attribute+}}^{oL}$ relation	<b>FULL OUTER JOIN</b>
relation $\bowtie_{\text{attribute+}}^{oR}$ relation	<b>LEFT OUTER JOIN</b>
projection ::=	
expression	<b>expression, can be just an attribute</b>
expression $\rightarrow$ attribute	<b>rename projected expression AS</b>
aggregationexp ::=	
aggregation( * attribute )	<b>without renaming</b>
aggregation( * attribute ) $\rightarrow$ attribute	<b>with renaming AS</b>
expression, condition, aggregation, attribute ::=	
<i>as in SQL, but excluding subqueries</i>	

# SQL

```
statement ::=
    CREATE TABLE tablename (
        * attribute type inlineconstraint*
        * [CONSTRAINT name]? constraint deferrable?
    ) ;
|
    DROP TABLE tablename ;
|
    INSERT INTO tablename tableplaces? values ;
|
    DELETE FROM tablename
    ? WHERE condition ;
|
    UPDATE tablename
    SET setting+
    ? WHERE condition ;
|
    query ;
|
    CREATE VIEW viewname
    AS ( query ) ;
|
    ALTER TABLE tablename
+   alteration ;
|
    COPY tablename FROM filepath ;
    ## postgresql-specific, tab-separated

query ::=
    SELECT DISTINCT? columns
    ? FROM table+
    ? WHERE condition
    ? GROUP BY attribute+
    ? HAVING condition
    ? ORDER BY attributeorder+
|
    query setoperation query
|
    query ORDER BY attributeorder+
    ## no previous ORDER in query
|
    WITH localdef+ query

table ::=
    tablename
|   table AS? tablename ## only one iteration allowed
|   ( query ) AS? tablename
|   table jointype JOIN table ON condition
|   table jointype JOIN table USING (attribute+)
|   table NATURAL jointype JOIN table

condition ::=
    expression comparison compared
|   expression NOT? BETWEEN expression AND expression
|   condition boolean condition
|   expression NOT? LIKE 'pattern*'
|   expression NOT? IN values
|   NOT? EXISTS ( query )
|   expression IS NOT? NULL
|   NOT ( condition )

type ::=
    CHAR ( integer ) | VARCHAR ( integer ) | TEXT
    | INT | FLOAT

inlineconstraint ::=      ## not separated by commas!
    PRIMARY KEY
    | REFERENCES tablename ( attribute ) policy*
    | UNIQUE | NOT NULL
    | CHECK ( condition )
    | DEFAULT value

constraint ::=
    PRIMARY KEY ( attribute+ )
    | FOREIGN KEY ( attribute+ )
    REFERENCES tablename ( attribute+ ) policy*
    | UNIQUE ( attribute+ ) | NOT NULL ( attribute )
    | CHECK ( condition )

policy ::=
    ON DELETE|UPDATE CASCADE|SET NULL

deferrable ::=
    NOT? DEFERRABLE (INITIALLY DEFERRED|IMMEDIATE)?

tableplaces ::=
    ( attribute+ )

values ::=
    VALUES ( value+ ) ## VALUES only in INSERT
    | ( query )

setting ::=
    attribute = value

alteration ::=
    ADD COLUMN attribute type inlineconstraint*
    | DROP COLUMN attribute

localdef ::=
    WITH tablename AS ( query )

columns ::=
    * ## literal asterisk
    | column+

column ::=
    expression
    | expression AS name

attributeorder ::=
    attribute (DESC|ASC)?

setoperation ::=
    UNION | INTERSECT | EXCEPT

jointype ::=
    LEFT|RIGHT|FULL OUTER?
    | INNER?

comparison ::=
    = | < | > | <> | <= | >=
```

```

expression ::=
    attribute
    | tablename.attribute
    | value
    | expression operation expression
    | aggregation ( DISTINCT? *|attribute)
    | ( query )

value ::=
    integer | float | string ## string in single quotes
    | value operation value
    | NULL

boolean ::=
    AND | OR

## triggers

functiondefinition ::=
    CREATE FUNCTION functionname() RETURNS TRIGGER AS $$
    BEGIN
    * triggerstatement
    END
    $$ LANGUAGE 'plpgsql'
    ;

triggerdefinition ::=
    CREATE TRIGGER triggername
    whentriggerved
    FOR EACH ROW|STATEMENT
    ? WHEN ( condition )
    EXECUTE PROCEDURE functionname
    ;

whentriggerved ::=
    BEFORE|AFTER events ON tablename
    | INSTEAD OF events ON viewname

events ::= event | event OR events
event ::= INSERT | UPDATE | DELETE

triggerstatement ::=
    IF ( condition ) THEN statement+ elsif* END IF ;
    | RAISE EXCEPTION 'message' ;
    | statement ; ## INSERT, UPDATE or DELETE
    | RETURN NEW|OLD|NULL ;

elsif ::= ELSIF ( condition ) THEN statement+

compared ::=
    expression
    | ALL|ANY values

operation ::=
    "+" | "-" | "*" | "/" | "%"
    | "||"

pattern ::=
    % | _ | character ## match any string/char
    | [ character* ]
    | [^ character* ]

aggregation ::=
    MAX | MIN | AVG | COUNT | SUM

## privileges

statement ::=
    GRANT privilege+ ON object TO user+ grantoption?
    | REVOKE privilege+ ON object FROM user+ CASCADE?
    | REVOKE GRANT OPTION FOR privilege
    ON object FROM user+ CASCADE?
    | GRANT rolename TO username adminoption?

privilege ::=
    SELECT | INSERT | DELETE | UPDATE | REFERENCES
    | ALL PRIVILEGES ## | ...

object ::=
    tablename (attribute)+ | viewname (attribute)+
    | trigger ## | ...

user ::= username | rolename | PUBLIC

grantoption ::= WITH GRANT OPTION

adminoption ::= WITH ADMIN OPTION

## transactions

statement ::=
    START TRANSACTION mode* | BEGIN | COMMIT | ROLLBACK

mode ::=
    ISOLATION LEVEL level
    | READ WRITE | READ ONLY

level ::=
    SERIALIZABLE | REPEATABLE READ | READ COMMITTED
    | READ UNCOMMITTED

## indexes

statement ::=
    CREATE INDEX indexname ON tablename (attribute+)?

```

## JSON

Both `json*` and `member*` indicate comma-separated lists. Strings are in double-quotes, numbers use decimal dot.

```
json ::= object | array | string | number | boolean
object ::= "{" member* "}"
member ::= string ":" json
array ::= "[" json* "]"
```

**JSON Path:** Expressions are built from operators, the result is an array with all matching json elements. The syntax below is from Postgres JSON Paths, using `..` instead of `.**` and `?[(condition)]` instead of `?(condition)` is also allowed.

`$` is the path for the root of the document

`.` is the child access operator (e.g. `$.name` gives the value of the name attribute of the root node)

`.*` is the wild-card access operator, it selects all attribute values of an object, or all items in an array

`**` is the recursive descent operator (e.g. `$.**.name` gives the value of the name attribute of all objects in the document)

`[n]` is array indexing (n is an integer)

`[*]` is the wild-card indexing operator, it selects all items in arrays

`[a,b,c]` selects multiple attributes (in double quotes) or array indexes

`?(condition)` is used to filter values

`@` is the current object in conditions (`$.*?(@.x>1)` gets attributes of the root node whose x attribute exceeds 1)

**JSON Schema:** Each schema is a JSON document.

false matches nothing

true matches everything (same as `{}`)

Objects contain any number of keywords (as keys), that limit what is accepted. Keywords and types of values:

- `"enum"` (array) accepts only the listed values.
- `"type"` (string) accepts only the given type, one of object/array/string/number/integer/boolean.
- `"minimum", "maximum", "minLength", "maxLength", "minProperties", "maxProperties", "minItems", "maxItems"` (integer) specifies bounds for numbers, string lengths, array lengths and number of attributes respectively.
- `"properties"` (object with name:schema pairs) specifies schemas for attributes of objects.  
E.g. `{"properties":{"x":{"type":"string"}, "y":false}}` accepts only objects where the type of attribute "x" is a string (or "x" does not exist) and attribute "y" does not exist.
- `"additionalProperties"` (schema) specifies the schema for all attributes not mentioned in "properties".
- `"required"` (array of strings) accepts only objects that have all the listed attributes
- `"items"` (schema) accepts only arrays where all items are accepted by the given schema
- `"contains"` (schema) accepts only arrays that where at least one item is accepted by the given schema
- `"uniqueItems"` (boolean) if boolean is true, accepts only arrays where items are unique
- `"allOf", "anyOf", "oneOf"` (array of schemas) accepts only what is accepted by all of, at least one of, or exactly one of the given schemas.
- `"not"` (schema) accepts only what is not accepted by the given schema.
- `"definitions"` (object with name:schema pairs) specifies named schemas, that can be used with `"$ref"`. Only used in the root object of a schema.
- `"$ref"` (string) accepts values that are accepted by the referenced schema. Use `"#"` to refer back to the root of the schema. Use `"#\definitions\x"` to refer to definition "x".