Real-World Relativity: The GPS Navigation System

People often ask me "What good is Relativity?" It is a commonplace to think of Relativity as an abstract and highly arcane mathematical theory that has no consequences for everyday life. This is in fact far from the truth.

Consider for a moment that when you are traveling in a commercial airliner, the pilot and crew are navigating to your destination with the aid of data from the Global Navigation Satellite System (GNSS), of which the United States NAVSTAR Global Positioning System (GPS for short) is the most familiar component. In fact, "GPS" is often synonymous with satellite navigation, even it is now one of three global satellite navigation systems in operation along with the Russian GLONASS and EU Galileo satellite systems (they will be joined by the Chinese BeiDou-2 system when it expands to global scale in the early 2020s), While this article is specifically about NAVSTAR GPS, the basic operating principles are similar across the various GNSS implementations.

GPS was developed by the United States Department of Defense to provide a satellite-based navigation system for the U.S. military. It was later put under joint DoD and Department of Transportation control to provide for both military and civilian navigation uses, and has become a part of daily life. Most recent-model cars are equipped with built-in GPS navigation systems (increasingly as standard equipment), you can purchase hand-held GPS navigation units that will give you your position on the Earth (latitude, longitude, and altitude) to an accuracy of 5 to 10 meters that weigh only a few ounces and cost around $100, and GPS technology is increasingly found in smartphones (though not all smartphones derive location information from GPS satellites).

The nominal GPS configuration consists of a network of 24 satellites in high orbits around the Earth, but up to 30 or so satellites may be on station at any given time. Each satellite in the GPS constellation orbits at an altitude of about 20,000 km from the ground, and has an orbital speed of about 14,000 km/hour (the orbital period is roughly 12 hours - contrary to popular belief, GPS satellites are not in geosynchronous or geostationary orbits). The satellite orbits are distributed so that at least 4 satellites are always visible from any point on the Earth at any given instant (with up to 12 visible at one time). Each satellite carries with it an atomic clock that "ticks" with a nominal accuracy of 1 nanosecond (1 billionth of a second). A GPS receiver in an airplane determines its current position and course by comparing the time signals it receives from the currently visible GPS satellites (usually 6 to 12) and trilaterating on the known positions of each satellite[1]. The precision achieved is remarkable: even a simple hand-held GPS receiver can determine your *absolute* position on the surface of the Earth to within 5 to 10 meters in only a few seconds. A

GPS receiver in a car can give accurate readings of position, speed, and course in real-time!

More sophisticated techniques, like Differential GPS (DGPS) and Real-Time Kinematic (RTK) methods, deliver centimeter-level positions with a few minutes of measurement. Such methods allow use of GPS and related satellite navigation system data to be used for high-precision surveying, autonomous driving, and other applications requiring greater real-time position accuracy than can be achieved with standard GPS receivers.

To achieve this level of precision, the clock ticks from the GPS satellites must be known to an accuracy of 20-30 nanoseconds. However, because the satellites are constantly moving relative to observers on the Earth, effects predicted by the Special and General theories of Relativity must be taken into account to achieve the desired 20-30 nanosecond accuracy.

Because an observer on the ground sees the satellites in motion relative to them, Special Relativity predicts that we should see their clocks ticking more slowly (see the Special Relativity lecture). Special Relativity predicts that the on-board atomic clocks on the satellites should fall behind clocks on the ground by about 7 microseconds per day because of the slower ticking rate due to the time dilation effect of their relative motion [2].

Further, the satellites are in orbits high above the Earth, where the curvature of spacetime due to the Earth's mass is less than it is at the Earth's surface. A prediction of General Relativity is that clocks closer to a massive object will seem to tick more slowly than those located further away (see the Black Holes lecture). As such, when viewed from the surface of the Earth, the clocks on the satellites appear to be ticking *faster* than identical clocks on the ground. A calculation using General Relativity predicts that the clocks in each GPS satellite should get ahead of ground-based clocks by 45 microseconds per day.

The combination of these two relativitic effects means that the clocks on-board each satellite should tick faster than identical clocks on the ground by about 38 microseconds per day (45-7=38)! This sounds small, but the high-precision required of the GPS system requires nanosecond accuracy, and 38 microseconds is 38,000 nanoseconds. If these effects were not properly taken into account, a navigational fix based on the GPS constellation would be false after only 2 minutes, and errors in global positions would continue to accumulate at a rate of about 10 kilometers each day! The whole system would be utterly worthless for navigation in a very short time.

The engineers who designed the GPS system included these relativistic effects when they designed and deployed the system. For example, to counteract the General Relativistic effect once on orbit, the onboard clocks were designed to "tick" at a slower frequency than ground reference clocks, so that once they were in their proper orbit stations their clocks would appear to tick at about the correct rate as compared to the reference atomic clocks at the GPS ground stations. Further, each GPS receiver has built into it a microcomputer that, in addition to performing the calculation of position using 3D trilateration, will also compute any additional special relativistic timing calculations required [3], using data provided by the satellites.

Relativity is not just some abstract mathematical theory: understanding it is absolutely essential for our global navigation system to work properly!

For more information about GPS, see

> NAVSTAR Global Positioning System Joint Project Office website
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> GPS FAQ provided by the FAA.

Notes

> [1] - Trilateration is a method of determining positions using the points of intersection of three overlapping circles or spheres. It differs from the more familiar method of **Triangulation** in that it does not use measurements of angles.
> [Thanks to Lt. Matthew Mosher, USN at USSTRATCOM for suggesting this note.]
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> [2] - *Relativity and the Global Positioning System*, Neil Ashby, 2002, Physics Today, May 2002, 41. Ashby does the calculations I cite for the time differences due to special and general relativitic effects. If citing this article for scholarly work regarding these numbers, please cite Ashby's article as one possibility.
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> I get more questions on relativity and GPS than any other web essay I've written. Since I wrote the original, a number of hard-to-find sources available only on paper are being scanned and made available in web-accessible form. In particular, this excellent 1996 article by Henry Fliegel and Raymond DiEspositi, online from the US Naval Observatory as a PDF document: GPS and Relativity: An Engineering Overview. Similarly, the short paper by Weiss and Ashby in 1997 (same Ashby as the 2002 Physics Today article) is also available from the USNO: GPS Receivers and Relativity.
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> [3] - While the primary general relativistic correction is taken care of on-board by the design clock frequency before launch and does not need to be computed by an individual receiver, the special relativistic corrections that require knowledge of the orbital parameters of the specific GPS satellites whose

signals are being measured are not. As described in the GPS Interface Control Document ICD-GPS-200C (10 Oct 1993), applying these corrections is the responsibility of the user's equipment (Section 20.3.3.3.3.1, "User Algorithm for SV Clock Correction"). The calculations are relatively straightforward and require very little beyond basic arithmetic, and make use of information transmitted in the data packets that come down from each spacecraft. [Thanks to Luca Rep who wrote asking about which corrections were done on the user's equipment.]

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