MATEMATIK
Chalmers Tekniska Hogskola

Dugga

Fourieranalys/Fourier Metoder 2020

Namn och personnummer:

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1. (1P) How many linearly independent solutions are there which satisfy:

$$u_t = ku_{xx}, \quad t > 0, \quad x \in (0, \ell), \quad u(0,t) = 0, \quad u_x(\ell, t) = 0?$$

   (a) there are no solutions
   (b) there is precisely one solution
   (c) there are infinitely many solutions
   (d) there are precisely 42 solutions

2. (1P) Consider the following problem:

$$\begin{cases} u_{tt}(x,t) - u_{xx}(x,t) = 0 & 0 < t,\ 0 < x < 1 \\ u(x,0) = \sin(\pi x) & x \in [0,1] \\ u_t(x,0) = 0 & x \in [0,1] \\ u(0,t) = 0 = u(1,t) & t > 0 \end{cases}$$

   How can we solve this problem?

   (a) take the Fourier transform of the PDE in the x variable
   (b) solve a regular Sturm-Liouville problem and take the Laplace transform in the t variable
   (c) use separation of variables, superposition and a Fourier series
   (d) none of these will work

3. (1P) The Fourier series of the function which is equal to e^x on the interval $[-\pi, \pi]$ is

$$\sum_{n\in\mathbb{Z}} \frac{(-1)^n \sinh(\pi^2)}{\pi(\pi - in)} e^{inx}.$$

   Compute

$$\sum_{n\in\mathbb{Z}} \frac{\sinh(\pi^2)}{\pi(\pi - in)}.$$

   (a) the series does not converge
   (b) the series converges to e^π
   (c) the series converges to $\frac{e^\pi + e^{-\pi}}{2}$
   (d) none of these are correct

4. (1P) Let

$$c_n := \frac{1}{2\pi} \int_{-\pi}^{\pi} x e^{-inx} dx.$$

Compute

$$\sum_{n \in \mathbb{Z}} |c_n|^2.$$

(a) the series does not converge

(b) the series converges to 2π

(c) the series converges to π^3

(d) none of these are correct

5. (1P) Let

$$c_n := \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin(x) e^{-inx}.$$

Compute:

$$\lim_{n \to \infty} c_n n^{42}.$$

(a) the limit does not exist

(b) the limit exists but there is insufficient information to compute its value

(c) the limit exists and is equal to 1

(d) none of these are correct