

Övning 1

Review of algebra

50 Förenkla $\frac{2x^2 - 3x - 2}{x^2 - 4}$ ($x \neq \pm 2$)

$$2x^2 - 3x - 2 = 0 \quad \text{pq-formel}$$

$$x^2 - \frac{3}{2}x - 1 = 0 \Rightarrow x = \frac{3}{4} \pm \sqrt{\left(\frac{3}{4}\right)^2 + 1} = \frac{3}{4} \pm \sqrt{\frac{9}{16} + 1} = \frac{3}{4} \pm \sqrt{\frac{25}{16}}$$
$$= \frac{3}{4} \pm \frac{5}{4}$$

$$\Rightarrow x_1 = -\frac{1}{2}, \quad x_2 = 2$$

$$\Rightarrow 2x^2 - 3x - 2 = 2\left(x + \frac{1}{2}\right)(x - 2) = (2x + 1)(x - 2)$$

$$x^2 - 4 = (x - 2)(x + 2)$$

$$\Rightarrow \frac{2x^2 - 3x - 2}{x^2 - 4} = \frac{(2x + 1)(x - 2)}{(x - 2)(x + 2)} = \frac{2x + 1}{x + 2}, \quad x \neq \pm 2$$

Extra uppgift: Lös olikheterna: a) $\frac{x}{x+1} > 1$, b) $|-2x+1| \leq 3$

Lösn. a) $\frac{x}{x+1} > 1 \Leftrightarrow \frac{x}{x+1} - 1 > 0 \Leftrightarrow \frac{x - (x+1)}{x+1} > 0 \quad (x \neq -1)$

$$\Leftrightarrow \frac{x - x - 1}{x+1} > 0 \Leftrightarrow \frac{-1}{x+1} > 0.$$

-1 är negativ. För att $\frac{-1}{x+1}$ blir positiv måste $x+1$

vara negativ $\Rightarrow x+1 < 0 \Rightarrow x < -1$

$$\Rightarrow x \in (-\infty; -1)$$

eller tabell:

		-1	
-1		-	-
$x+1$	-	0	+
	+	0	-

$$\begin{aligned}
 b) \quad | -2x + 1 | &\leq 3 \quad (\Leftrightarrow) \quad -3 \leq -2x + 1 \leq 3 \quad \downarrow -1 \\
 &\Leftrightarrow \quad -4 \leq -2x \leq 2 \\
 &\Leftrightarrow \quad 2 \geq x \geq -1 \quad \downarrow \cdot \left(-\frac{1}{2}\right) \\
 &\Leftrightarrow \quad x \in [-1, 2]
 \end{aligned}$$

Review of analytic geometry

39. $x^2 + y^2 - 4x + 10y + 13 = 0$

$$\Leftrightarrow x^2 - 4x + y^2 + 10y + 13 = 0$$

Kvadratkomplettering

$$\Leftrightarrow x^2 - 2(2x) + 4 + y^2 + 2 \cdot 5y + 25 - 4 - 25 + 13 = 0$$

$$\Leftrightarrow (x - 2)^2 + (y + 5)^2 - 16 = 0$$

$$\Leftrightarrow (x - 2)^2 + (y + 5)^2 = 4^2$$

Cirkel med centrum $(2; -5)$ och radie $r = 4$.

42 $3x - 5y + 19 = 0$ $10x + 6y - 50 = 0$

$$\downarrow$$

$$-5y = -3x - 19$$

$$y = \frac{3}{5}x + \frac{19}{5}$$

$\uparrow$
 m_1

$$\downarrow$$

$$6y = -10x + 50$$

$$y = -\frac{10}{6}x + \frac{50}{6}$$

$$= -\frac{5}{3}x + \frac{25}{3}$$

$\uparrow$
 m_2

$$m_1 \cdot m_2 = \frac{3}{5} \cdot \left(-\frac{5}{3}\right) = -1 \Rightarrow \text{linjerna \u00e4r vinkelr\u00e4ta.}$$

Sk\u00e4rningspunkt:

$$\begin{aligned}
 \begin{cases} y = \frac{3}{5}x + \frac{19}{5} \\ y = -\frac{5}{3}x + \frac{25}{3} \end{cases} &\Rightarrow \frac{3}{5}x + \frac{19}{5} = -\frac{5}{3}x + \frac{25}{3} \Rightarrow \left(\frac{3}{5} + \frac{5}{3}\right)x = \frac{25}{3} - \frac{19}{5} = \frac{68}{15} \\
 &\Rightarrow \frac{34}{15}x = \frac{68}{15} \Rightarrow x = \frac{68}{34} = 2 \Rightarrow y = \frac{3}{5} \cdot 2 + \frac{19}{5} = \frac{25}{5} = 5
 \end{aligned}$$

$(2; 5)$