Övning 2:

s. 680 14 $100x^2 + 36y^2 = 225$

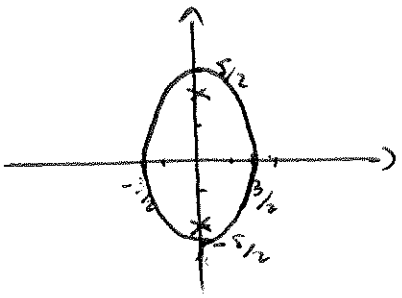
$$\Leftrightarrow \frac{100}{225}x^2 + \frac{36}{225}y^2 = 1 \quad (\Leftrightarrow \frac{x^2}{\left(\frac{225}{100}\right)} + \frac{y^2}{\left(\frac{225}{36}\right)} = 1$$

$$\Leftrightarrow \frac{x^2}{\left(\frac{15}{10}\right)^2} + \frac{y^2}{\left(\frac{15}{6}\right)^2} = 1 \quad \Leftrightarrow \frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

$b = \frac{5}{2} > \frac{3}{2} = a \Rightarrow$ transversala punkter är $(0, \pm \frac{5}{2})$.

$c^2 = b^2 - a^2 = \frac{25}{4} - \frac{9}{4} = \frac{16}{4} = 4 \Rightarrow c = \pm 2$

$\Rightarrow$ brännpunkter är $(0, \pm 2)$



Ex. Bestäm definitionsmängden till:

(a) $f(x) = \frac{1}{1-\sqrt{x-2}}$ (b) $g(x) = \sqrt{2-\sqrt{x}}$. (c) $h(x) = \frac{1}{1-\cos x}$

a) D_f: $x - 2 \geq 0 \Rightarrow x \geq 2$

OCH $1 - \sqrt{x-2} \neq 0 \Rightarrow \sqrt{x-2} \neq 1 \Rightarrow x - 2 \neq 1 \Rightarrow x \neq 3$.

$\Rightarrow D_f = [2, 3) \cup (3, \infty)$

b) $2 - \sqrt{x} \geq 0 \Rightarrow 2 \geq \sqrt{x} \Rightarrow 4 \geq x$

OCH

$x \geq 0$.

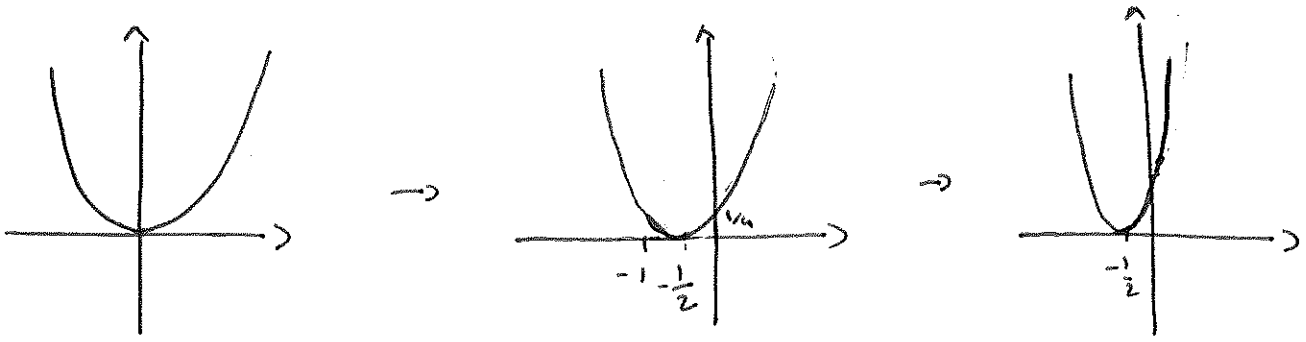
$\Rightarrow D_g = [0, 4]$

(c) $1 - \cos x \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 2k\pi \quad k \in \mathbb{Z}$.

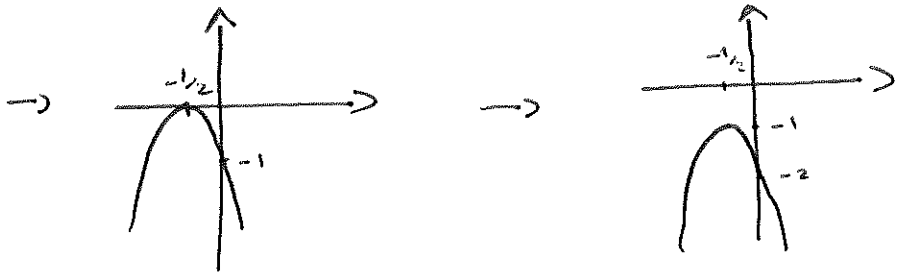
$\Rightarrow D_h = \mathbb{R} - \{2k\pi\}_{k \in \mathbb{Z}}$.

Ex. Rita grafen till funktionen $y = -(2x+1)^2 - 1$.

$$y = -(2x+1)^2 - 1 = -\left(2\left(x+\tfrac{1}{2}\right)\right)^2 - 1 = -4\left(x+\tfrac{1}{2}\right)^2 - 1$$



$y = x^2$ → $y = \left(x+\frac{1}{2}\right)^2$ → $y = 4\left(x+\frac{1}{2}\right)^2$ (stretch)



$y = -4\left(x+\frac{1}{2}\right)^2$ → $y = -4\left(x+\frac{1}{2}\right)^2 - 1$

Ex. 38 s. 44 $\quad f(x) = \dfrac{x}{1+x}$ och $g(x) = \sin(2x)$ $\quad \begin{cases} D_f = \mathbb{R} - \{-1\} \\ D_g = \mathbb{R} \end{cases}$

(a) $f \circ g(x) = f(g(x)) = f(\sin 2x) = \dfrac{\sin 2x}{1+\sin 2x}$

$1 + \sin(2x) \neq 0 \Rightarrow \sin(2x) \neq -1 \Rightarrow 2x \neq -\frac{\pi}{2} + 2k\pi \Rightarrow x \neq -\frac{\pi}{4} + k\pi$

(b) $g \circ f(x) = g(f(x)) = g\left(\dfrac{x}{1+x}\right) = \sin\dfrac{2x}{1+x}$

$1+x \neq 0 \Rightarrow x \neq -1 \Rightarrow D_{g\circ f} = \mathbb{R} - \{-1\}$

(c) $f \circ f(x) = f(f(x)) = f\left(\dfrac{x}{1+x}\right) = \dfrac{\frac{1}{1+x}}{1+\frac{1}{1+x}} = \dfrac{\frac{1}{1+x}}{\frac{1+x+x}{1+x}} = \dfrac{1}{1+2x}$, $\quad x \neq -1$, $x \neq -\frac{1}{2}$

$D_{f\circ f} = \mathbb{R} - \{-1, -\frac{1}{2}\}$.

(d) $g \circ g(x) = g(g(x)) = g(\sin 2x) = \sin(2(\sin 2x)) \quad D_{g\circ g} = \mathbb{R}$.