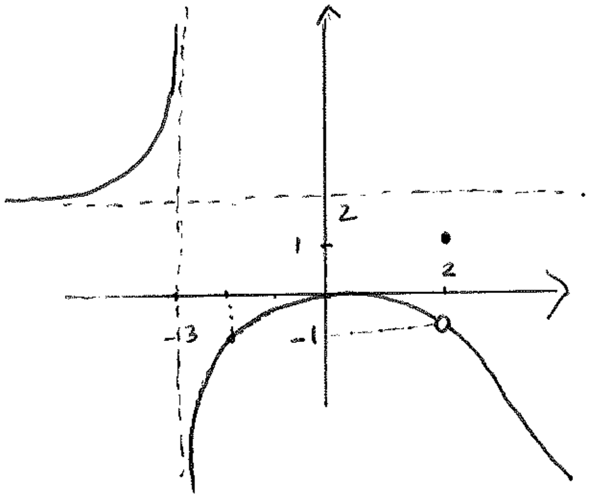
Övning 4

1. Ex.



Bestäm:

$D_f = (-\infty, -3) \cup (-3, \infty)$

$V_f = (-\infty, 0] \cup \{1\} \cup (2, \infty)$

$\lim_{x \to -\infty} f(x) = 2 \to y = 2$ asymptot

$\lim_{x \to \infty} f(x) = -\infty$

$$\left.\begin{array}{l} \lim_{x \to -3^-} f(x) = \infty \\ \lim_{x \to -3^+} f(x) = -\infty \end{array}\right\} x = -3 \text{ asymptot}$$

$\lim_{x \to 2} f(x) = -1$

$f(2) = 1$

2. Beräkna $\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{x+3} - 2}$ om det existerar.

Lösn: $\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{x+3} - 2} = \frac{1-1}{2-2} = \frac{0}{0}$ obestämt.

$$\frac{x^2-1}{\sqrt{x+3}-2} = \frac{(x^2-1)(\sqrt{x+3}+2)}{(\sqrt{x+3}-2)(\sqrt{x+3}+2)} = \frac{(x-1)(x+1)(\sqrt{x+3}+2)}{x+3-4}$$

$$= \frac{\cancel{(x-1)}(x+1)(\sqrt{x+3}+2)}{\cancel{x-1}} = (x+1)(\sqrt{x+3}+2)$$

$$\Rightarrow \lim_{x \to 1} \frac{x^2-1}{\sqrt{x+3}-2} = \lim_{x \to 1} (x+1)(\sqrt{x+3}+2) = 2(\sqrt{4}+2) = 2(2+2) = 8.$$

3. Beräkna $\lim_{x \to a^+} \frac{|x-a|}{x^2 - a^2}$

(*) $|x-a| = x-a$ eftersom $x > a$ $(x - a > 0)$.

Lösn. $\lim_{x \to a^+} \frac{|x-a|}{x^2-a^2} \overset{(*)}{=} \lim_{x \to a^+} \frac{x-a}{(x-a)(x+a)} = \lim_{x \to a^+} \frac{1}{x+a} = \frac{1}{2a}$

4. $$f(x) = \begin{cases} \dfrac{x+3}{x^2+7x+12} & x \geq -3 \\ \dfrac{2x+6}{|x+3|} & x < -3. \end{cases}$$

(a) Beräkna $\lim\limits_{x \to -3^+} f(x)$ och $\lim\limits_{x \to -3^-} f(x)$

(b) existerar $\lim\limits_{x \to -3} f(x)$?

Lösn. (a). $$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} \frac{x+3}{x^2+7x+12} = \lim_{x \to -3^+} \frac{x+3}{(x+3)(x+4)}$$

$$= \lim_{x \to -3^+} \frac{1}{x+4} = \frac{1}{-3+4} = 1$$

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{2x+6}{|x+3|} = \lim_{x \to -3^-} \frac{2(x+3)}{-(x+3)} = -2.$$

$\big(|x+3| = -(x+3)$ då $x < -3$ $(x+3 < 0)\big)$.

(b) $\lim\limits_{x \to -3^-} f(x) \neq \lim\limits_{x \to -3^+} f(x) \Rightarrow \lim\limits_{x \to -3} f(x)$ existerar inte.

Ex. 32 s. 103 Beräkna $$\lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{\frac{1}{x^2} - \frac{1}{x^2}}{0} = \frac{0}{0}.$$

$$\frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{x^2 - (x+h)^2}{x^2(x+h)^2} = \frac{(x-(x+h))(x+(x+h))}{x^2(x+h)^2} = \frac{(x-x-h)(x+x+h)}{x^2(x+h)^2}$$

$$= \frac{-h(2x+h)}{x^2(x+h)^2}$$

$$\Rightarrow \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{\frac{-h(2x+h)}{x^2(x+h)^2}}{h} = \lim_{h \to 0} \frac{-\cancel{h}(2x+1)}{x^2(x+h)^2} \cdot \frac{1}{\cancel{h}} = \frac{-2x}{x^4} = -\frac{2}{x^3}$$