MMA630 Project 1

Monte-Carlo simulations of an Ornstein–Uhlenbeck process

Introduction

In this project, the focus lies on the simulation of an Ornstein–Uhlenbeck process, defined as the solution of the SDE

$$X_t = x_0 - a \int_0^t X_s \,\mathrm{d}s + \sigma W_t,$$

on the time interval $[0, T]$, with $T < +\infty$. This process is interpreted as a mean-reverting process, where $a > 0$ will affect the speed of the mean-reversion, $\sigma > 0$ is the volatility of the process, $\{W_t : t \in [0, T]\}$ is a standard Wiener process, and x_0 is a deterministic initial value. The main goal of the project is to efficiently and accurately approximate the expected value, $\mathbb{E}[X_T]$, at time T, where $X := \{X_t : t \in [0, T]\}$ denotes the Ornstein–Uhlenbeck process.

Main tasks

- Implement an Euler–Maruyama scheme to numerically simulate a path for X.
- Use the Euler–Maruyama implementation to simulate M paths of X and approximate $\mathbb{E}[X_T]$ by a Monte-Carlo estimator $E_M[X_T]$.
- Implement a multilevel Monte-Carlo method to approximate $\mathbb{E}[X_T]$, and compare the efficiency with the standard Monte-Carlo method.

Report

Hand in a clear and concise report on the project. The report should include:

1. **Introduction:**
   - General introduction to the project.
2. **Theory:**
   - The necessary theory regarding the equation. Existence and uniqueness results, analytic expressions for expectation and variance.
   - Definition of the Euler–Maruyama scheme, and corresponding strong/weak convergence results for the equation.
   - Monte-Carlo simulation. Convergence of the Monte-Carlo estimator.
   - Derivation of the multilevel Monte-Carlo method. Error convergence rate and choice of number of simulations on each level.

   Make sure that you introduce the theory for the specific example and do not copy the literature. Do not forget to give references when you are using existing results.
3. **Implementation:**
   - Description of details regarding the implementation.
4. **Numerical examples:**

- Plotted solutions for different mesh sizes with respect to the same path of the Brownian motion.
- Convergence plots (weak/strong, MC/MLMC) with reference lines.
- Time stamps for a given TOL on error for the different methods.

Deadline: February 17, 2020, 07:00
Requirement: You can receive up to four bonus points on the project that are valid on this year's exams.
Formalities: Your project has to be submitted before the deadline via canvas in order to qualify for bonus points.
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