# MMA630 Project 2

Simulation of an Ornstein–Uhlenbeck process via the Feynman–Kac formula

## Introduction

We consider the Ornstein–Uhlenbeck process from Project 1, i.e.

$$X_t = x_0 - a \int_0^t X_s \, \mathrm{d}s + \sigma W_t, \tag{1}$$

on the time interval $[0, T]$, with $T < +\infty$. In the previous project, this process was simulated for a given initial value $x_0 \in \mathbb{R}$ by means of Monte-Carlo methods. Here we shall instead look into the Feynman–Kac formula of this process, i.e. the corresponding parabolic PDE, and solve it for numerous values of $x \in [-R, R] \subset \mathbb{R}$ simultaneously. The main goal of the project lies in deriving this PDE and solving it using the finite element method.

## Main tasks

- Derive the parabolic PDE that represents the Ornstein–Uhlenbeck process (1) by using the Feynman–Kac formula and localize it to $[-R, R]$.
- Apply the finite element method to solve the derived equation numerically.

## Report

Hand in a clear and concise report on the project. The report should include:

1. **Introduction:**
   - General introduction to the project.
2. **Theory:**
   - Definition of the Feynman–Kac formula.
   - Derivation of the resulting PDE, written as an initial value problem.
   - Derivation of corresponding variational formulation.
   - Localization to an interval $[-R, R]$ of finite length and using Dirichlet boundary conditions.
   - Construction of your FE-space and definition of your FEM problem.

   Make sure that you introduce the theory for the specific example and do not copy the literature. Do not forget to give references when you are using existing results.
3. **Implementation:**
   - Computation of matrix elements for the FEM matrix system.
   - Description of details regarding the implementation.
4. **Numerical examples:**

- Plotted solutions for the PDE corresponding to $\mathbb{E}[X_T]$ on the computational domain for some different time stamps (e.g. $t = 1, 2, .., T$) in one plot to see how the solution behaves over time. Use linear finite elements.

5. **Discussion/Conclusion:**

   - Discussion regarding the differences between simulation of the process using MC-methods and the Feynman–Kac approach.
   - Pros and cons, when it is useful to apply which method, etc.

**Deadline:** March 12, 2020, 07:00
**Requirement:** You can receive up to 4 bonus points on the project that are valid on this year's exams.
**Formalities:** Your project has to be submitted before the deadline via canvas in order to qualify for bonus points.
**Teachers:** Annika Lang and Per Ljung