# Support Vector Machines and Kernel methods

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# Reference

The content and the slides are adapted from

S. Rogers and M. Girolami, A First Course in Machine Learning (FCML), 2nd edition, Chapman & Hall/CRC 2016, ISBN: 9781498738484

# Classification syllabus

- 4 classification algorithms.
- Of which:
  - 2 are probabilistic.
    - Bayes classifier.
    - Logistic regression.
  - 2 non-probabilistic.
    - K-nearest neighbours.
    - Support Vector Machines (SVM).
- There are many others!

# Topics ...

- Linear SVM
- Soft-Margin SVM
- Kernels - Kernel SVM
- Classifier Performance

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- **Linear SVM**
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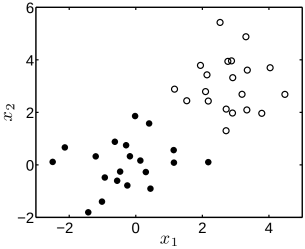
- We have seen several algorithms where we find the parameters that optimise something:
  - Minimise the loss.
  - Maximise the likelihood.
  - Maximise the posterior (MAP).
- The Support Vector Machine (SVM) is no different:
- It finds the *decision boundary* that maximises the margin.

# Some data

- We'll 'think' in 2-dimensions.

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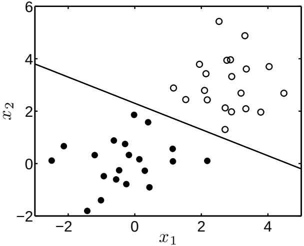


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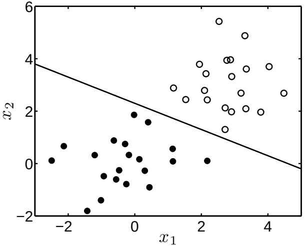


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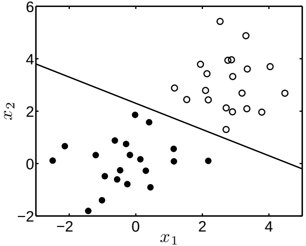


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$$\mathbf{w}^\mathsf{T}\mathbf{x} + b = 0$$

- Our task is to find $\mathbf{w}$ and $b$
- Once we have these, classification is easy:

$$\begin{aligned} \mathbf{w}^\mathsf{T}\mathbf{x}_{\text{new}} + b > 0 \quad &: \quad t_{\text{new}} = 1 \\ \mathbf{w}^\mathsf{T}\mathbf{x}_{\text{new}} + b < 0 \quad &: \quad t_{\text{new}} = -1 \end{aligned}$$

- i.e. $t_{\text{new}} = \text{sign}(\mathbf{w}^\mathsf{T}\mathbf{x}_{\text{new}} + b)$

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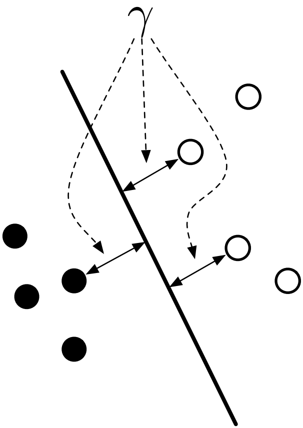

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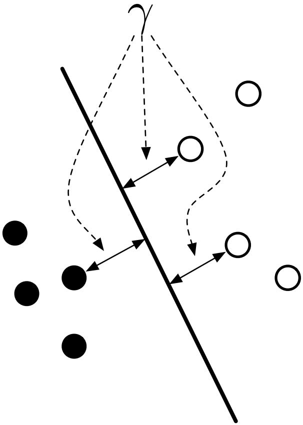
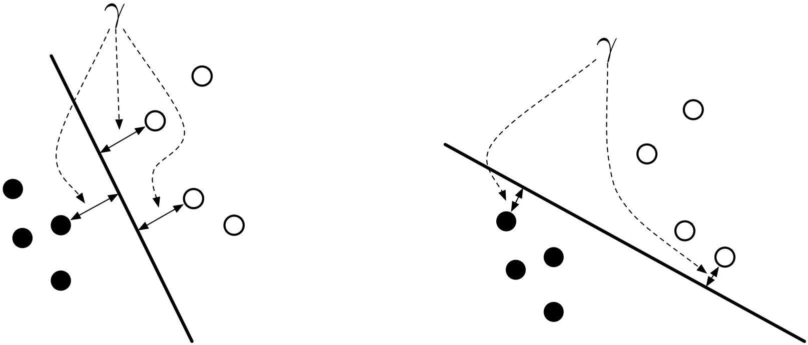



Perpendicular distance from the decision boundary to the closest points on each side.

# Why maximise the margin?

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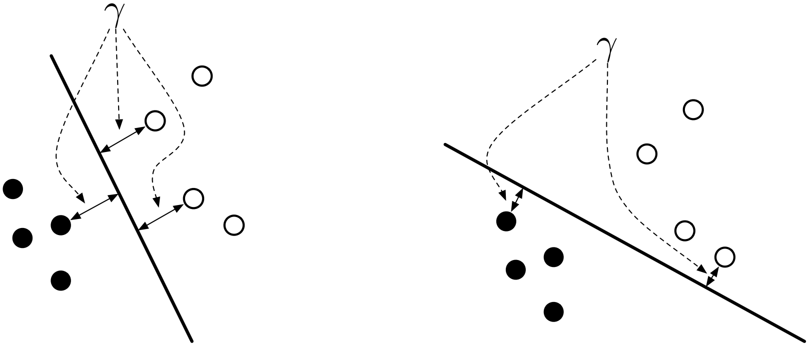
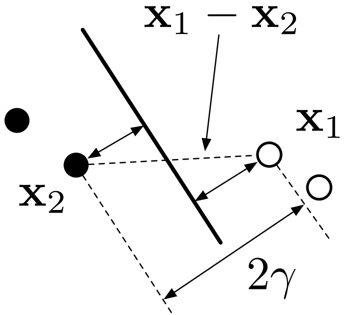



- Maximum margin decision boundary (left) seems to better reflect the data characteristics than other boundary (right).
- Note how margin is much smaller on right and closest points have changed.
- There is going to be one ‘best’ boundary (w.r.t margin)
- Statistical theory justifying the choice.

# Computing the margin

$$2\gamma = \frac{1}{||\mathbf{w}||}\mathbf{w}^{\mathsf{T}}(\mathbf{x}_1 - \mathbf{x}_2)$$



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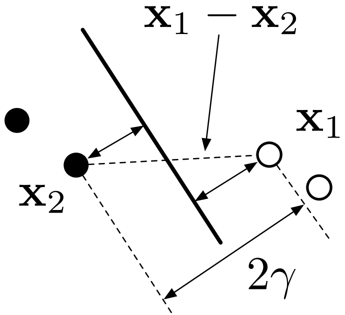

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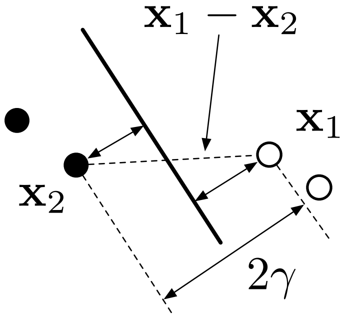
Therefore:

$$\begin{aligned} (\mathbf{w}^T\mathbf{x}_1 + b) - (\mathbf{w}^T\mathbf{x}_2 + b) &= 2 \\ \mathbf{w}^T(\mathbf{x}_1 - \mathbf{x}_2) &= 2 \\ \gamma = \frac{1}{||\mathbf{w}||} \end{aligned}$$



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- There are some constraints:
  - For $\mathbf{x}_n$ with $t_n = 1$: $\mathbf{w}^\mathsf{T}\mathbf{x}_n + b \geq 1$
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- Which can be expressed more neatly as:

$$t_n(\mathbf{w}^\mathsf{T}\mathbf{x}_n + b) \geq 1$$

- (This is why we use $t_n = \pm 1$ and not $t_n = \{0, 1\}$.)

# Maximising the margin

- We have the following optimisation problem:

$$\underset{\mathbf{w}}{\operatorname{argmin}} \ \frac{1}{2}\mathbf{w}^{\mathsf{T}}\mathbf{w}$$

$$\text{Subject to: } t_n(\mathbf{w}^{\mathsf{T}}\mathbf{x}_n + b) \geq 1$$

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- Can put the constraints into the minimisation using Lagrange multipliers:

$$\underset{\mathbf{w}}{\operatorname{argmin}} \frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{w} - \sum_{n=1}^{N} \alpha_n(t_n(\mathbf{w}^\mathsf{T}\mathbf{x}_n + b) - 1)$$

$$\text{Subject to: } \alpha_n \geq 0$$

# What now?

- Let's think about what happens at the solution (we'll see why...)
- We know that $\frac{\partial}{\partial \mathbf{w}} = 0$ and $\frac{\partial}{\partial b} = 0$.

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$$\begin{aligned}
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\frac{\partial}{\partial b} &= -\sum_n \alpha_n t_n = 0
\end{aligned}$$

- From which we can infer that:

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- Substitute these back into our optimisation problem:

$$
\begin{aligned}
& \frac{1}{2}\mathbf{w}^{\mathsf{T}}\mathbf{w} - \sum_n \alpha_n (t_n(\mathbf{w}^{\mathsf{T}}\mathbf{x}_n + b) - 1) \\
& \vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
= \quad & \sum_n \alpha_n - \frac{1}{2}\sum_{n,m} \alpha_n \alpha_m t_n t_m \mathbf{x}_n^{\mathsf{T}}\mathbf{x}_m
\end{aligned}
$$

$$\frac{1}{2}\mathbf{w}^\mathsf{T}\mathbf{w} - \sum_n \alpha_n(t_n(\mathbf{w}^\mathsf{T}\mathbf{x}_n + b) - 1)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$= \sum_n \alpha_n - \frac{1}{2}\sum_{n,m} \alpha_n\alpha_m t_n t_m \mathbf{x}_n^\mathsf{T}\mathbf{x}_m$$

- Instead of minimising the previous expression, we can maximise this one (for reasons we won't go into).
- Subject to:

$$\alpha_n \geq 0$$
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\alpha_n \geq 0
$$

$$
\sum_n \alpha_n t_n = 0
$$

- Decision function was $\text{sign}(\mathbf{w}^\mathsf{T}\mathbf{x}_{\text{new}} + b)$ and is now:

$$
t_{\text{new}} = \text{sign}\left(\sum_{n=1}^{N} \alpha_n t_n \mathbf{x}_n^\mathsf{T}\mathbf{x}_{\text{new}} + b\right)
$$

# So?

$$\underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{n.m=1}^{N} \alpha_n \alpha_m t_n t_m \mathbf{x}_n^{\mathsf{T}} \mathbf{x}_m$$

$$\text{subject to} \quad \sum_{n=1}^{N} \alpha_n t_n = 0, \quad \alpha_n \geq 0$$

- This is a standard optimisation problem (quadratic programming)
- Has a single, global solution. This is very useful!
- Many algorithms around to solve it.
- e.g. `quadprog` in Matlab...

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- Once we have $\alpha_n$:

$$t_{\text{new}} = \text{sign}\left( \sum_{n=1}^{N} \alpha_n t_n \mathbf{x}_n^{\mathsf{T}} \mathbf{x}_{\text{new}} + b \right)$$

# Primal and Dual

## Primal

$$\underset{\mathbf{w}}{\operatorname{argmin}} \frac{1}{2}\mathbf{w}^{\mathsf{T}}\mathbf{w}$$

$$\text{Subject to: } t_n(\mathbf{w}^{\mathsf{T}}\mathbf{x}_n + b) \geq 1$$

## Dual

$$\underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{n.m=1}^{N} \alpha_n \alpha_m t_n t_m \mathbf{x}_n^{\mathsf{T}} \mathbf{x}_m$$

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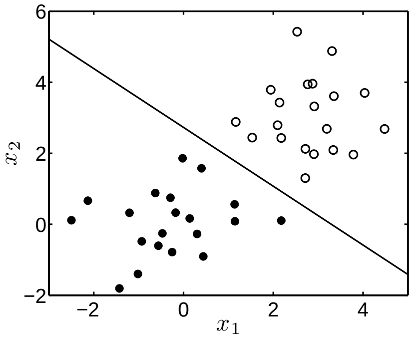
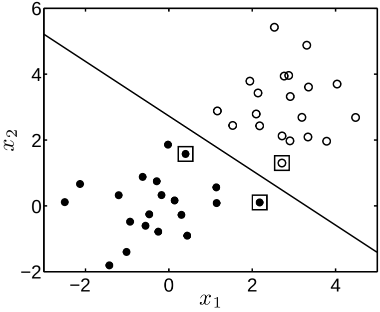
# Optimal boundary




- Optimisation gives us $\alpha_1, \ldots, \alpha_N$
- Compute $\mathbf{w} = \sum_n \alpha_n t_n \mathbf{x}_n$
- Compute $b = t_n - \mathbf{w}^\mathsf{T}\mathbf{x}_n$ (for one of the closest points)
  - Recall that we defined $\mathbf{w}^\mathsf{T}\mathbf{x}_n + b = \pm 1 = t_n$ for closest points.
- Plot $\mathbf{w}^\mathsf{T}\mathbf{x} + b = 0$

# Support Vectors

- At the optimum, only 3 non-zero $\alpha$ values (squares).




- $t_{\mathsf{new}} = \mathsf{sign}\left(\sum_n \alpha_n t_n \mathbf{x}_n^{\mathsf{T}} \mathbf{x}_{\mathsf{new}} + b\right)$
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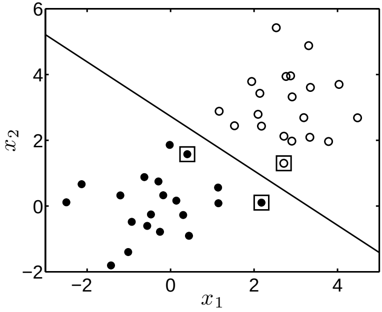


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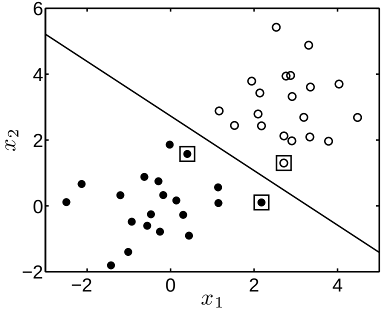


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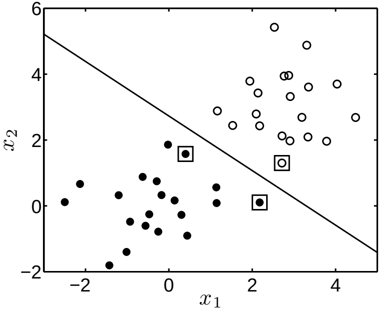


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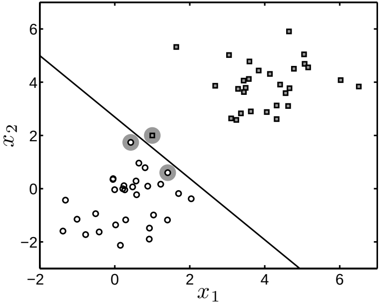
- Predictions only depend on these data-points!
- We knew that – margin is only a function of closest points.
- These are called Support Vectors
- Normally a small proportion of the data:
  - Solution is *sparse*.

# Is sparseness good?

- Not always:



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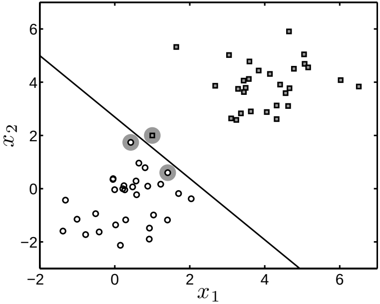


- Why does this happen?

$$t_n(\mathbf{w}^\mathsf{T}\mathbf{x}_n + b) \geq 1$$

- All points must be on correct side of boundary.
- This is a *hard margin*

# Topics ...

- Linear SVM
- **Soft-Margin SVM**
- Kernels - Kernel SVM
- Classifier Performance

# Soft margin

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- And when we add Lagrange etc:

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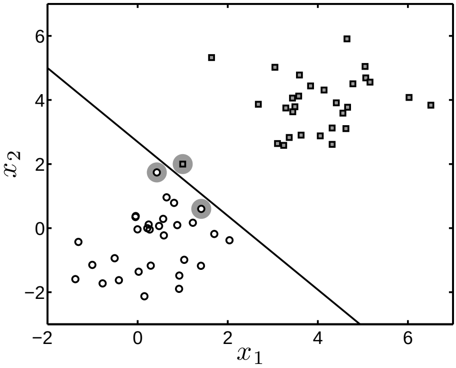
$$\text{subject to} \;\; \sum_{n=1}^{N}\alpha_n t_n = 0, \;\; 0 \leq \alpha_n \leq C$$

- The **only** change is an upper-bound on $\alpha_n$!

# Soft margins

- Here's our problematic data again:

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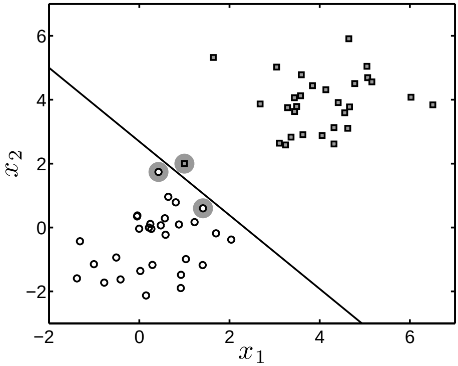
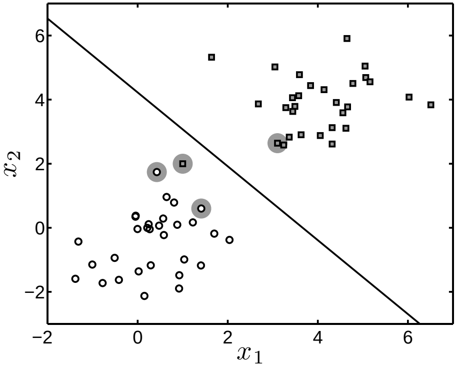



- $\alpha_n$ for the 'bad' square is 3.5.
- So, if we set $C < 3.5$, we should see this point having less influence and the boundary moving to somewhere more sensible...

# Soft margins

- Try $C = 1$




- We have an extra support vector.
- And a better decision boundary.

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  - ...and lose any sparsity.

- Choose it using cross-validation.

# SVMs – some observations

- In our example, we started with 3 parameters:

$$\mathbf{w} = [w_1, w_2]^\mathsf{T}, \quad b$$

- In general: D+1.

# SVMs – some observations

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$$\mathbf{w} = [w_1, w_2]^\mathsf{T}, \quad b$$

- In general: D+1.
- We now have $N$: $\alpha_1, \ldots, \alpha_N$

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- In our example, we started with 3 parameters:

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- In general: D+1.
- We now have $N$: $\alpha_1, \ldots, \alpha_N$
- Sounds harder?
- Depends on data dimensionality:
  - Typical Microarray dataset:
  - $D \sim 3000$, $N \sim 30$.
  - In some cases $N \ll D$

# Topics ...

- Linear SVM
- Soft-Margin SVM
- **Kernels - Kernel SVM**
- Classifier Performance

# Inner products

- Here's the optimisation problem:

$$\underset{\boldsymbol{\alpha}}{\text{argmax}} \sum_n \alpha_n - \frac{1}{2} \sum_{n,m} \alpha_n \alpha_m t_n t_m \mathbf{x}_n^\mathsf{T} \mathbf{x}_m$$

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$$t_{\text{new}} = \text{sign}\left( \sum_n \alpha_n t_n \mathbf{x}_n^\mathsf{T} \mathbf{x}_{\text{new}} + b \right)$$

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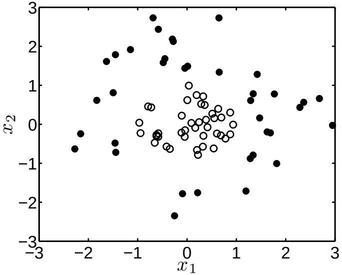
- Data ($\mathbf{x}_n, \mathbf{x}_m, \mathbf{x}_{\text{new}}$, etc) only appears as inner (dot) products:

$$\mathbf{x}_n^\mathsf{T} \mathbf{x}_m, \;\; \mathbf{x}_n^\mathsf{T} \mathbf{x}_{\text{new}}, \text{etc}$$

# Projections

- Our SVM can find linear decision boundaries.
- What if the data requires something nonlinear?



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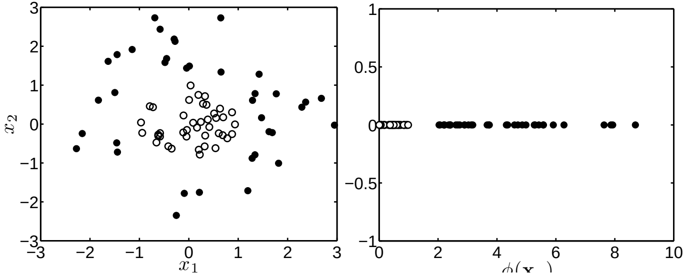


- We can transform the data e.g.:

$$\phi(\mathbf{x}_n) = x_{n1}^2 + x_{n2}^2$$

- So that it can be separated with a straight line.
- And use $\phi(\mathbf{x}_n)$ instead of $\mathbf{x}_n$ in our optimisation.

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- Our optimisation is now:

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- We can think of the dot product in the projected space as a function of the original data.

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- Polynomial:
$$k(\mathbf{x}_n, \mathbf{x}_m) = (1 + \mathbf{x}_n^{\mathsf{T}} \mathbf{x}_m)^{\beta}$$
- These all correspond to $\phi(\mathbf{x}_n)^{\mathsf{T}} \phi(\mathbf{x}_m)$ for some transformation $\phi(\mathbf{x}_n)$.
- Don't know what the projections $\phi(\mathbf{x}_n)$ are – don't need to know!

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- The optimisation is just as simple, regardless of the kernel choice.
  - Still a quadratic program.
  - Still a single, global optimum.
- We can find very complex decision boundaries with a linear algorithm!

# A technical point

- Our decision boundary was defined as $\mathbf{w}^\mathsf{T}\mathbf{x} + b = 0$.
- Now, $\mathbf{w}$ is defined as:

$$\mathbf{w} = \sum_{n=1}^{N} \alpha_n t_n \phi(\mathbf{x}_n)$$

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- We **only know** $\phi(\mathbf{x}_n)^\mathsf{T}\phi(\mathbf{x}_m) = k(\mathbf{x}_n, \mathbf{x}_m)$
- So, we can't compute $\mathbf{w}$ or the boundary!
- But we can evaluate the predictions on a grid of $\mathbf{x}_{\text{new}}$ and use Matlab to draw a contour:

$$\sum_{n=1}^{N} \alpha_n t_n k(\mathbf{x}_n, \mathbf{x}_{\text{new}}) + b$$

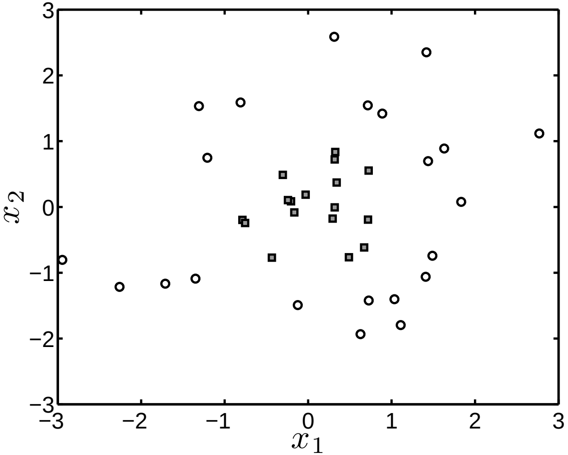
# Aside: kernelising other algorithms

- **Many** algorithms can be kernelised.
  - Any that can be written with data only appearing as inner products.
- Simple algorithms can be used to solve very complex problems!
- Class exercise:
  - KNN requires the distance between $\mathbf{x}_{\text{new}}$ and each $\mathbf{x}_n$:

$$(\mathbf{x}_{\text{new}} - \mathbf{x}_n)^\mathsf{T}(\mathbf{x}_{\text{new}} - \mathbf{x}_n)$$

  - Can we kernelise it?

# Example – nonlinear data

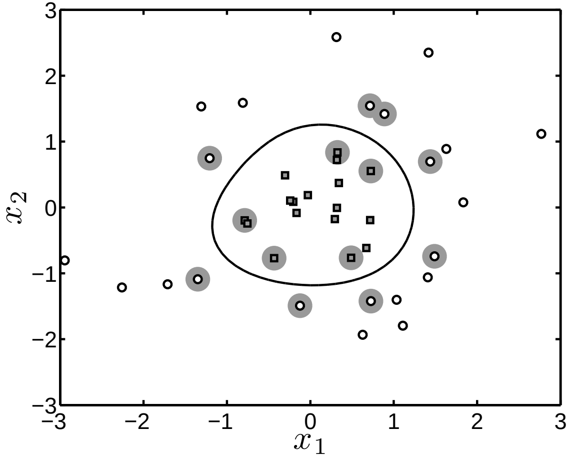



- We'll use a Gaussian kernel:

$$k(\mathbf{x}_n, \mathbf{x}_m) = \exp\left\{-\beta(\mathbf{x}_n - \mathbf{x}_m)^{\mathsf{T}}(\mathbf{x}_n - \mathbf{x}_m)\right\}$$

- And vary $\beta$ ($C = 10$).

# Examples




- $\beta = 1$.

$$k(\mathbf{x}_n, \mathbf{x}_m) = \exp\left\{-\beta(\mathbf{x}_n - \mathbf{x}_m)^{\mathsf{T}}(\mathbf{x}_n - \mathbf{x}_m)\right\}$$

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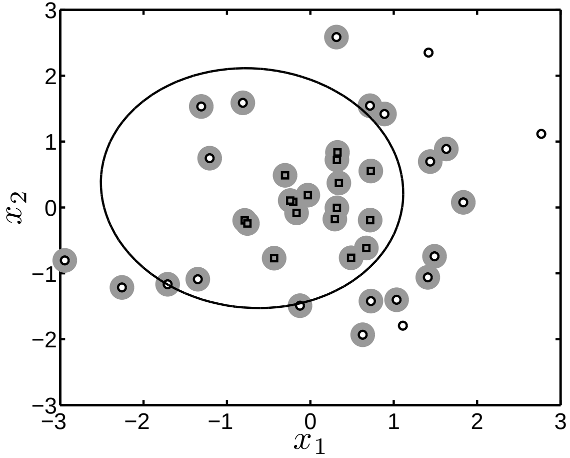


- $\beta = 0.01$.

$$k(\mathbf{x}_n, \mathbf{x}_m) = \exp\left\{-\beta(\mathbf{x}_n - \mathbf{x}_m)^{\mathsf{T}}(\mathbf{x}_n - \mathbf{x}_m)\right\}$$

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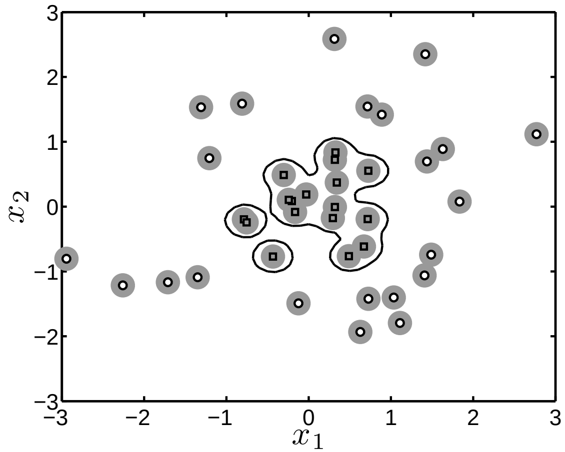


- $\beta = 50$.

$$k(\mathbf{x}_n, \mathbf{x}_m) = \exp\left\{-\beta(\mathbf{x}_n - \mathbf{x}_m)^{\mathsf{T}}(\mathbf{x}_n - \mathbf{x}_m)\right\}$$

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- $\beta = 50$ was too complex:
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- $\beta = 1$ was about right.
- Neither $\beta = 50$ or $\beta = 0.01$ will *generalise* well.
- Both are also non-sparse (lots of support vectors).

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- Cross-validation!
- Search over $\beta$ and $C$
  - SVM scales with $N^3$ (naive implementation)
  - For large $N$, cross-validation over many $C$ and $\beta$ values is infeasible.

# Summary - SVMs

- Described a classifier that is optimised by maximising the *margin*.
- Did some re-arranging to turn it into a quadratic programming problem.
- Saw that data only appear as inner products.
- Introduced the idea of kernels.
- Can fit a linear boundary in some other space without explicitly projecting.
- Loosened the SVM constraints to allow points on the wrong side of boundary.
- Other algorithms can be kernelised...we'll see a clustering one in the future.

# Topics ...

- Linear SVM
- Soft-Margin SVM
- Kernels - Kernel SVM
- **Classifier Performance**

# Performance evaluation

- We've seen 4 classification algorithms.
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  - Which algorithm?
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- Need performance indicators.
- We'll cover:
  - 0/1 loss.
  - ROC analysis (sensitivity and specificity)
  - Confusion matrices

# 0/1 loss

- 0/1 loss: proportion of times classifier is wrong.
- Consider a set of predictions $t_1, \ldots, t_N$ and a set of true labels $t_1^*, \ldots, t_N^*$.
- Mean loss is defined as:

$$\frac{1}{N}\sum_{n=1}^{N} \delta(t_n \neq t_n^*)$$

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- ($\delta(a)$ is 1 if $a$ is true and 0 otherwise)
- Advantages:
  - Can do binary or multiclass classification.
  - Simple to compute.
  - Single value.

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- Assume only 1% of population is diseased.
- Diseased: $t = 1$
- Healthy: $t = 0$
- What if we always predict healthy? ($t = 0$)
- Accuracy 99%
- But classifier is rubbish!

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- **False negatives (FN)** – the number of objects with $t_n^* = 1$ that are classified as $t_n = 0$ (diseased people diagnosed as healthy).

# Sensitivity

$$S_e = \frac{TP}{TP + FN}$$

- The proportion of diseased people that we classify as diseased.
- The higher the better.
- In our example, $S_e = 0$.

# Specificity

$$S_p = \frac{TN}{TN + FP}$$

- The proportion of healthy people that we classify as healthy.
- The higher the better.
- In our example, $S_p = 1$.

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# Optimising sensitivity and specificity

- We would like both to be as high as possible.
- Often increasing one will decrease the other.
- Balance will depend on application:
- e.g. diagnosis:
  - We can probably tolerate a decrease in specificity (healthy people diagnosed as diseased)....
  - ...if it gives us an increase in sensitivity (getting diseased people right).

# ROC analysis

- Many classification algorithms involve setting a threshold.
- e.g. SVM:

$$t_{\text{new}} = \text{sign}\left(\sum_{n=1}^{N} t_n \alpha_n k(\mathbf{x}_n, \mathbf{x}_{\text{new}}) + b\right)$$

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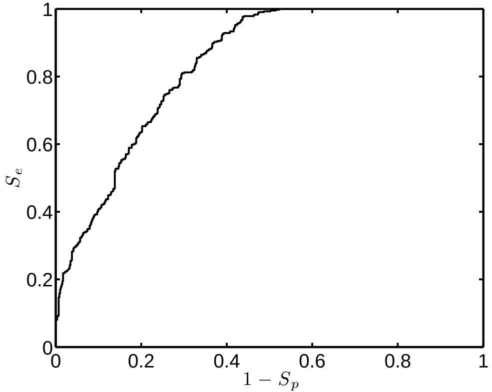
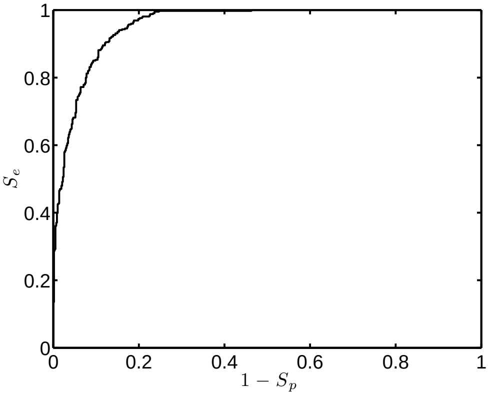
- Implies a threshold of zero (sign function)
- However, we could use any threshold we like....
- The Receiver Operating Characteristic (ROC) curve shows how $S_e$ and $1 - S_p$ vary as the threshold changes.

# ROC curve



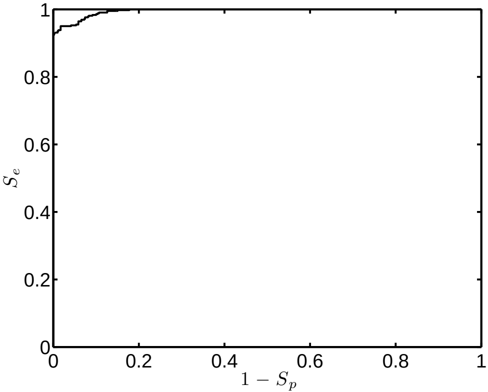

- SVM for nonlinear data with $\beta = 50$.
- Each point is a threshold value.
  - Bottom left – everything classified as 0 (-1 in SVM)
  - Top right – everything classified as 1.
- Goal: get the curve to the top left corner – perfect classification ($S_e = 1, S_p = 1$).

# ROC curve




- SVM for nonlinear data with $\beta = 0.01$.
- Better than $\beta = 50$
  - Closer to top left corner.

# ROC curve




- SVM for nonlinear data with $\beta = 1$.
- Better still.

# AUC

- We can quantify performance by computing the Area Under the ROC Curve (AUC)
- The higher this value, the better.
  - $\beta = 50$: AUC=0.8348
  - $\beta = 0.01$: AUC= 0.9551
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- AUC is generally a safer measure than 0/1 loss.

# Confusion matrices

The quantities we used to compute $S_e$ and $S_p$ can be neatly summarised in a table:

| | | True class 1 | True class 0 |
|---|---|---|---|
| Predicted class | 1 | TP | FP |
| Predicted class | 0 | FN | TN |

- This is known as a confusion matrix
- It is particularly useful for multi-class classification.
- Tells us where the mistakes are being made.
- Note that normalising columns gives us $S_e$ and $S_p$

## Confusion matrices – example

- 20 newsgroups data.
- Thousands of documents from 20 classes (newsgroups)
- Use a Naive Bayes classifier ($\approx$ 50000 dimensions (words)!)
  - Details in book Chapter.
- $\approx$ 7000 independent test documents.
- Summarise results in $20 \times 20$ confusion matrix:

True class

| Predicted class | | ... | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | ... | 4 | 2 | 0 | 2 | 10 | 4 | 7 | 1 | 12 | 7 | 47 |
| | 2 | ... | 0 | 0 | 4 | 18 | 7 | 8 | 2 | 0 | 1 | 1 | 3 |
| | 3 | ... | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 4 | ... | 1 | 0 | 1 | 28 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | ⋮ | | | | | | |
| | 16 | ... | 3 | 2 | 2 | 5 | 17 | 4 | 376 | 3 | 7 | 2 | **68** |
| | 17 | ... | 1 | 0 | 9 | 0 | 3 | 1 | 3 | 325 | 3 | **95** | 19 |
| | 18 | ... | 2 | 1 | 0 | 2 | 6 | 2 | 1 | 2 | 325 | 4 | 5 |
| | 19 | ... | 8 | 4 | 8 | 0 | 10 | 21 | 1 | 16 | 19 | **185** | 7 |
| | 20 | ... | 0 | 0 | 1 | 0 | 1 | 1 | 2 | 4 | 0 | 1 | **92** |

| | | True class | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ... | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 18 | 19 | 20 |
| Predicted class 1 | ... | 4 | 2 | 0 | 2 | 10 | 4 | 7 | 1 | 12 | 7 | 47 |
| 2 | ... | 0 | 0 | 4 | 18 | 7 | 8 | 2 | 0 | 1 | 1 | 3 |
| 3 | ... | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | ... | 1 | 0 | 1 | 28 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| ⋮ | | | | | | | | | | | | |
| 16 | ... | 3 | 2 | 2 | 5 | 17 | 4 | 376 | 3 | 7 | 2 | **68** |
| 17 | ... | 1 | 0 | 9 | 0 | 3 | 1 | 3 | 325 | 3 | **95** | 19 |
| 18 | ... | 2 | 1 | 0 | 2 | 6 | 2 | 1 | 2 | 325 | 4 | 5 |
| 19 | ... | 8 | 4 | 8 | 0 | 10 | 21 | 1 | 16 | 19 | **185** | 7 |
| 20 | ... | 0 | 0 | 1 | 0 | 1 | 1 | 2 | 4 | 0 | 1 | **92** |

- Algorithm is getting ‘confused’ between classes 20 and 16, and 19 and 17.
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| Predicted class | | True class | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ... | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 18 | 19 | 20 |
| | 1 | ... | 4 | 2 | 0 | 2 | 10 | 4 | 7 | 1 | 12 | 7 | 47 |
| | 2 | ... | 0 | 0 | 4 | 18 | 7 | 8 | 2 | 0 | 1 | 1 | 3 |
| | 3 | ... | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 4 | ... | 1 | 0 | 1 | 28 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | ⋮ | | | | | | |
| | 16 | ... | 3 | 2 | 2 | 5 | 17 | 4 | 376 | 3 | 7 | 2 | **68** |
| | 17 | ... | 1 | 0 | 9 | 0 | 3 | 1 | 3 | 325 | 3 | **95** | 19 |
| | 18 | ... | 2 | 1 | 0 | 2 | 6 | 2 | 1 | 2 | 325 | 4 | 5 |
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  - 16: talk.religion.misc
  - 20: soc.religion.christian

| | | True class | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ... | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 18 | 19 | 20 |
| Predicted class | 1 | ... | 4 | 2 | 0 | 2 | 10 | 4 | 7 | 1 | 12 | 7 | 47 |
| | 2 | ... | 0 | 0 | 4 | 18 | 7 | 8 | 2 | 0 | 1 | 1 | 3 |
| | 3 | ... | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 4 | ... | 1 | 0 | 1 | 28 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | ⋮ | | | | | | |
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| Predicted class \ True class | ... | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ... | 4 | 2 | 0 | 2 | 10 | 4 | 7 | 1 | 12 | 7 | 47 |
| 2 | ... | 0 | 0 | 4 | 18 | 7 | 8 | 2 | 0 | 1 | 1 | 3 |
| 3 | ... | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | ... | 1 | 0 | 1 | 28 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| ⋮ | | | | | | ⋮ | | | | | | |
| 16 | ... | 3 | 2 | 2 | 5 | 17 | 4 | 376 | 3 | 7 | 2 | **68** |
| 17 | ... | 1 | 0 | 9 | 0 | 3 | 1 | 3 | 325 | 3 | **95** | 19 |
| 18 | ... | 2 | 1 | 0 | 2 | 6 | 2 | 1 | 2 | 325 | 4 | 5 |
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- Confusion matrix helps us direct our efforts to improving the classifier.

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  - ROC/AUC
- Introduced confusion matrices – a way of assessing the performance of a multi-class classifier.